Motivating a Formalism for Phenomenologically Distinct Present:

An Inquiry in Physics and Logic of Time

Kartik Tiwari

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Contents

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1. **Introduction**

In this paper, I motivate a recursive formulation for the philosophically dense notion of *present* by setting up a formal system and compiling a set of valuable tools associated with temporal logic.

I start with describing a toy universe with one radioactive atom in it to familiarize the reader with the concept of world states. Though used in a significantly different context with fairly different tools, the origins of the Calculus of World States can be traced back to Carew A. Meredith (Rescher and Urquhart 1971). For the purposes of this paper, the World State Calculus describes any world with the truth values of a minimal set of propositions, which I attempt to present in the most general way. Then, I introduce a system of coding the truth values of such descriptive propositions into a natural number which has a unique decomposition that enables one to extract the string of truth values of propositions from its associated world state number. The modified idea of a clock-event proposition is then introduced to establish that every instance of time has a *unique* world-state number associated with it.

After presenting these basic tools, I take a brief detour to discuss some non-trivial subtleties associated with the discretization of time and the problem of unequal infinities arising from a branching or what I call a *generative* view of future.

Then, I return to the framework that was developed earlier to discuss the intuitive idea of the causal reach of an agent of change. The causal reach is then used to establish a notion of *present* that is distinct from other moments in the agent's first-person experience of time. Since the notion of present discussed is distinct in the agent's experience of time and does not correspond to an absolute characteristic embedded within time itself, I refer to it as a phenomenologically distinct present. An attempt to formalize the aforementioned formulation of *now* is attempted using algorithmic recursion.

Finally, I present some philosophical speculations about the compatibility of a recursive present within deterministic and stochastic views of future. Towards the end I present an important objection to my work which stems from a mathematical theorem and consider its repercussions.

> At various places in the paper, I make claims that are contentious. While making such claims, I shall use red color. The purpose of this is to urge the reader to engage with certain nuanced aspects of the subject deeper than what is presented in this paper.

2. Descriptions of the World that Vary with Time

2.1 *A Toy Universe*

Consider a toy model of a universe that consists of only a single Uranium atom. The Uranium atom spontaneously undergoes a radioactive decay and transforms into a form of Thorium. Consider a proposition schemata $P^{'}$ which becomes either true or false in the toy world depending on the value that variable t takes

$$P^{'} : \exists_t x Ux \text{ (where U(x) := x is a Uranium Atom)}$$

Note that the above is a proposition schemata which is neither true nor false, only its instances are. It is also important to disambiguate that a proposition of the form 'There exists a uranium atom at 3:00 pm' is tenseless and either eternally true or eternally false. The interpretation of the subscript t relevant to this paper is of the following form. At a specified time t, the proposition 'There exists a Uranium atom' is either true or false. Therefore, to talk about truth values in the context of $P^{'}$, we require a mapping between instances of time and truth value of the instances of proposition schemata.

Let us suppose the proposition $\exists x Ux$ holds true only until some time t_0, after which the Uranium atom radioactively decays. We express this idea using a function $W(P^{'})$ that acts on the proposition schemata $P^{'}$ and serves as a map between instances of time and truth values. In the case of the proposition described above, W function acts as follows

$$W(P^{'}) = \begin{cases} 1 & t < t_0 \\ 0 & t \geq t_0 \end{cases}$$

This can be visualized on a simple space with one axis as time $\hat{t}$ and another axis representing the world state at a given time $\hat{\omega}$ as demonstrated in Fig. 1.

At this stage, one may point out that a more complete description of our toy universe would instead require at least two proposition at any given time; an instance of the aforementioned $P^{'}$ and an instance of the following proposition schemata $P^{''}$

$$P^{''} : \exists_t x Tx \text{ (where T(x) := x is Thorium)}$$

As was the case earlier, we can create a mapping to meaningfully discuss the truth values associated

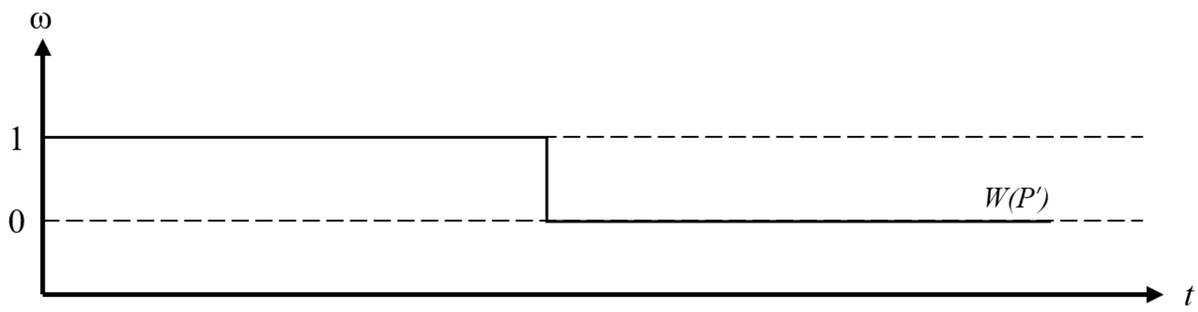

Figure 1: Modelling a toy universe on a simple world-state space

with $P^{''}$ using $W(P^{''})$

$$W(P^{''}) = \begin{cases} 0 & t < t_0 \\ 1 & t \geq t_0 \end{cases}$$

With two relevant propositions at each time, one can create strings of binary digits that represent the possible states of the world. Since instances of $P^{'}$) and $P^{''}$ can both take the value of 0 or 1, the possibilities can be represented by the following set $\{00, 01, 10, 11\}$ where the n^{th} digit from the left in the string represents the truth value of the instance of $P^{n'}$ proposition schemata at the concerned time.

Even after introducing two propositions for every time instance, one can point out that the description is incomplete without also introducing a proposition about the helium nucleus that is the byproduct of the radioactive decay. Or, going further down in the reductionist rabbit hole, we can have propositions about individual protons, neutrons and electrons in our toy universe. One can go even further, right to the edge of our understanding, and talk directly about the quarks and gluons and other fundamental particles in terms of formal propositions to have a description of our toy universe. It is clear that even for a toy model with only a single atom, the descriptions can get very sophisticated. However, for any given level of understanding that provides us with n unique propositions as a description of the universe at each time, we can create strings of n binary digits that express the possible world states.

After a first look at the toy universe, one might be inclined to claim that there is a physical constraint on our world in the form of

$$\Box(\exists x U x) \leftrightarrow (\forall x \neg T x)$$

where the necessity operator $\Box$ - *for this example* - has the semantic interpretation of 'at all times t'. Such a proposition ensures that as long as the Uranium atom exists, the Thorium atom cannot coexist with it at the same time. This constraint would rule out the possibility $\{11\}$. However, this constraint is also a descriptive proposition about physical nature of the universe and can be equivalently reformulated in terms of the following proposition schemata

$$P^{'''} : (\exists_t x U x) \leftrightarrow (\forall_t x \neg T x)$$

which has a mapping between time and truth value of the form $W(P^{'''}) = 1$. Claiming $W(P^{'''}) = 1$ for all t can only be a conjecture (since, logically, it is possible for an instance of $P^{'''}$ to be true or false without creating contradiction within the system). On the other hand, if our formal system had an axiom of the form $\Box((\exists x U x) \leftrightarrow (\forall x \neg T x))$ then the coexistence of Uranium or Thorium would would lead to a logical inconsistency. In that case, we would be justified in eliminating certain possibilities a priori. Therefore, the elimination of possible world states a priori can only be justified within the logical system via deduction from axioms and not via inference from descriptive propositions.

2.2 *Generalizing to Infinite Propositions*

Consider a world for which it takes infinite propositions at every instance of time to formulate a complete description. A possible world state at a specific time for such a universe can be given by an infinite string of the following kind

$$W(P^{'})W(P^{''})W(P^{'''})W(P^{''''}).......... := 1101010001110110.......$$

One might wonder how does one create such a string and compile the truth values of the infinite number of propositions in finite time. This line of thought leads to unnecessary confusion. One does not *compile* such a string. Just as how the world simply exists in a certain state at a given time, an isomorphic abstract representation of the world in the form of the binary string simply exists at that time as well. We might never be able to know which string of binary digits expresses the state of the world at any instance of time but it is not reason enough to discard the independent abstract existence of the aforementioned mapping of the world onto the binary string.

However, the generalization to infinite propositions does present a problem that we need to find a way around. The problem is that such a string renders it impossible for one to distinguish amongst infinitely many world states (which might even be infinitely different). To clarify, suppose two world states ω_1 and ω_2 contain the following leading terms

$$\omega_1 = 10100001001000101000010......$$

$$\omega_2 = 10100001001000101000010......$$

Even if we have checked for the first 100 million propositions and have found that the truth values for all such propositions are equal, we still would not be in a position to claim $\omega_1 = \omega_2$ because we would have infinitely many more propositions to compare. It becomes impossible to establish identity amongst world states if the length of the binary string is infinite. One might be tempted to discard this problem as an confusion of the earlier kind as well by claiming 'Even if it is impossible to establish identity amongst infinite binary strings, it is not reason enough to discard the independent abstract existence of such an identity mapping'. This is true but we cannot brush this problem under the carpet because the concept of change is central to the formulation of present that I am trying to motivate.

2.3 *Developing Unique World State Numbers*

One way to get around the problem of not being able to differentiate between changing world states, is to not compare the infinitely long strings; instead creating another abstract mapping that represents the world state not in the form of infinite strings. We develop the following Godel-eqsue mapping by exploiting the properties of prime factorization of natural numbers.

Consider the prime decomposition of an arbitrary number -

$$\mathbf{Z} = 2^a \times 3^b \times 5^c \times ... \times n^k$$

Using the infinitely long binary string, let us substitute the power of the n^{th} prime factor with the truth value of the n^{th} proposition. For example, consider the example of a world state where only instances of P'' and P''' are true and rest all proposition are false. The information about such a world state can be embedded in a composite number in the following manner

$$W(P^{'})W(P^{''})W(P^{'''})W(P^{''''})... \longrightarrow 011000... \longrightarrow 2^0.3^1.5^1.7^0.11^0.13^0... = \mathbf{15}$$

If we have to compare any other world state with the aforementioned one, instead of comparing the infinite digits of the binary strings, we can simply compare the world state numbers because had there been any difference between the two strings, the world-state number would have different prime factors.

It is clear that a large set of natural numbers would not form meaningful maps to the binary strings. For example, the number 4 does not have any meaningful decomposition in the suggested mapping because it is equivalent to 2^2 and a proposition can only posses a truth value from the set $\{0, 1\}$. Fortunately, this does not pose a problem to us at the moment because it can be demonstrated that the infinity of allowed world-state numbers is equal to the infinity of counting numbers (which is further equal to the infinite binary array).

I re-emphasize that - in cases of infinite propositions - we need not code the world states in such a composite number by mapping the infinitely long string onto the powers of infinite prime numbers. There simple exists an abstract mapping such that the world states at any time can be represented by a single composite number. From the completely unambiguous prime decomposition of the world state number, we can extract all the information about state of the world at that time.

Earlier, to identify a change in world states, we were forced to compare arrays of infinite digits. However, in the space of our world-state numbers, the comparison is drawn between natural numbers which - though obviously enormous - are made of finite number of digits.

3. **Clock Event Propositions and Global Entropy**

Colloquially speaking, the Second Law of Thermodynamics claims that measure of disorder (as denoted by a mathematically well defined entity named entropy) of the universe is always increasing. Before employing the concept of global entropy, we first develop an understanding of a clock event.

3.1 *Clock Event Postulate*

A Clock Event (sometimes known as *nominal*) is defined as an event that uniquely marks off an instance of time. For example, the birth of Issac Asimov is an event that can never happen in the future ever again, nor can one find an event identical to that at any other time in history. In other words, assuming the occurrence of that event fixes a particular unique time. Considering P to be a proposition about the occurrence of a Clock Event for some time a, then

$$(\exists_a P \wedge \exists_{a'} P) \leftrightarrow (a = a')$$

Clock Event Postulate is the stipulation that such a proposition that uniquely marks an instance of time, exists for each instance of time. In the notation that I am following, this proposition can be included in the logic by adding the following as a postulate

$$\mathbf{L}((\exists_t P \wedge \exists_{t'} P) \leftrightarrow (t = t'))$$

The $\mathbf{L}\alpha$ has the standard temporal logic semantic interpretation of 'Always α'. It can be decomposed as $\mathbf{L}\alpha := (\mathbf{H}\alpha \wedge \alpha \wedge \mathbf{G}\alpha)$, which means α *has always been true, is true and is always going to be true.*

An implication of such a postulate in the previously described framework is that each instance of time has a unique number associated with it because in the string of truth values of propositions, for every time t, there will be a unique proposition $c(t)$ that becomes true only at that time. If the truth value of each such $c(t)$ is mapped to a unique prime number, then every instance of time has an injective mapping that unique prime number. Note that each instance of time has a mapping to a worldstate number but not every worldstate number has an associated instance of time.

> The name 'Clock Event Proposition' and 'Clock Event Postulate', in my opinion, are misnomers and lead to confusion. The intuition that this formalization tries to capture is that at a given time of the day, the clock displays a configuration (of its hands or digits) that happens neither earlier in the day, nor later. Only at that instance of time would a clock show that specific configuration (or at least a 24 Hours version). However, the primary purpose of a clock is not to label instances of time but to try and divide time in equal intervals. The unique labelling of the intervals is secondary to the division objective. If the duration between two distinct configurations was to vary randomly, then the configurations of the clock would be considered futile.
> When we borrow the notion of labelling instances of time in most temporal logics, we do so without the introduction of a mechanism for consistent division of time. A *metric* is the formal tool that is used for marking consistent intervals in time (or in any space more generally). Introducing a *metric* in a logic has several non trivial physical, formal and philosophical considerations associated with it. It is clear, by only knowing two world-states numbers in our framework, one cannot deduce how 'close' or 'distant' in time they are. Because of the loaded sense in which the word 'clock' is used, it is important to explicitly mention that Clock Event Postulate can be introduced in a logic without the necessity to include the notion of a *metric* as well.

3.2 *Finding a Physical Basis for Clock Event Postulate*

Returning back to the Second Law, physics establishes that the entropy of the universe is always increasing. There are various different formulations of entropy in terms of thermodynamics, statistical mechanics or information theory but they can all be shown to be equivalent. Therefore, if a proposition of the kind 'The global entropy of the universe calculated by method Ω is Λ' holds true at some time t, then for all times before t the global entropy would be less than Λ and at all times after t the global entropy would be more than Λ. Thus, the aforementioned proposition only holds true at time t and never before or after. Using the method of using claims about entropy seems to be a promising candidate for establishing physical grounds for the Clock Event Postulate.

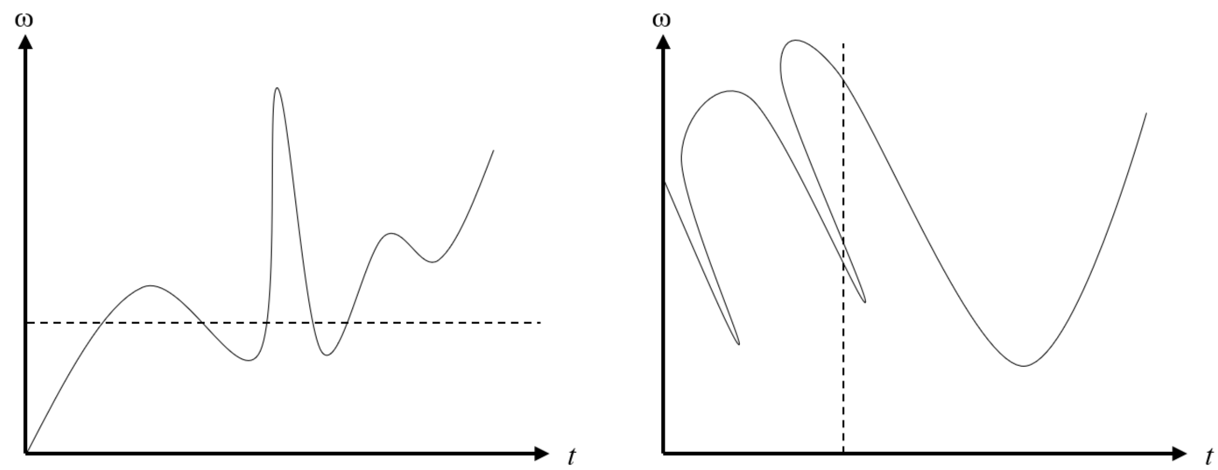

Figure 2: Certain curves on world-state space are a priori disallowed in our formalism. The figure on the left is such that the world is in the same state at multiple times but that is disallowed by clock event postulate. The figure on the right is such that the world is in multiple states at the same time but that is disallowed by the law of excluded middle.

> There are several nuances related to the Second Law that were ignored in this brief discussion. An important idea related to the Second Law is 'Poincaré Recurrence Theorem' that states certain dynamical systems will, after sufficiently long but finite time, return to their original state or one arbitrarily close to it. For example, consider a deck of cards with Black and Red suites separated neatly. Once shuffling starts, the deck of cards becomes more and more disordered. However, after an extremely large but finite number of shuffles the deck should return back to being ordered according to the Recurrence theorem. Even if one does not delve too deeply into the mathematics of recurrence theorem, the fact that entropy is a statistical and probabilistic concept should provide an intuition about how there is always an extremely low but non-zero probability of spontaneous decrease in entropy. Since Poincaré Recurrence time for macroscopic systems is much larger than the stipulated age of the universe itself, increase in entropy being a *local phenomena (in time)* is not considered a problem and Poincaré Recurrence is treated as only a mathematical inconvenience for our purposes.

3.3 *Discretization of Time*

As mentioned earlier, the possible world state numbers are an infinite subset of the infinite set of natural numbers (an infinite set is, in fact, defined by the property of its cardinality being equal to the cardinality of a proper subset of that set). Natural numbers form a countably infinite set (denoted by $\aleph_0$). On the other hand, the time axis is generally considered to be dense or continuous. This means for any time t and another time t'' ordered by the relation $t < t''$, there always exists another time instance t' such that the three instances can be ordered temporally with the precedence relation $t < t' < t''$ regardless of how close t and t'' are. Thus, time in physics is labelled by the set of Real Numbers and the cardinality of set of real numbers is denoted by $\aleph_1$.

We previously established using the clock event postulate that the mapping between the two axes is such that each world state number is associated with an unambiguously deconstructable time instance. At the current stage of our formulation, this is not possible. Cantor's diagonal slash argument establishes that the infinite set of real numbers is larger than the infinite set of natural numbers. Such a mapping from world-state axis to time axis is, therefore, not possible because there simply would not be enough possible world states to be allocated uniquely to the continuous time axis. (NOTE TO SELF: Include diagrams to clarify this properly. It becomes increasingly confusing the longer one stares at it)

To preserve the aforementioned mapping, one needs to discretize the time axis into some 'smallest' possible intervals of time. This goes against almost all of classical physics because performing calculus is only meaningful on functions that possess the mathematical property of continuity, rendering even the most basic differential equations (like Newton's Laws of Motion) broken. This view of calculus, though overwhelmingly mainstream, is not the only view that exists. There are forms of 'Calculus Without Limits' (some including epistemic considerations deriving from Nagarjuna's *Sunyavaad*) with which our discrete time axis would be compatible.

Further, there is another sense in which time axis is viewed as discrete from the perspective of modern physics. There is a limit to speed of causality imposed by Special Relativity (represented by c) and there is also a limit to the smallest possible meaningful length of space as governed by Quantum Mechanics (called the Planck Length). Since velocity is distance divided by time, using the speed of causality and Planck length, one can extract an interval of time called Planck Time. It is the amount of time that the fastest possible causation takes to travel the shortest possible length. Thus, there is no change possible lasting for less time than the Planck Time. Throughout that interval of time, the world *needs* to possess the same world-state number because no change can happen faster than the speed of causality and in a space shorter than the smallest possible length.

If there does exist a shortest meaningful interval of time which cannot be further divided, this has yet another philosophically loaded implication. Is one correct to claim that no time passes within the smallest possible interval? Intuitively, it seems reasonable to make claim that even within the shortest interval of time, from the beginning of that interval to the end of that interval, time has passed. However, this style of reasoning seems to stem from a confusion that draws a continuous axis of meta-time with values that change even within the interval of Planck Time. We have no physical basis to assume such an axis of meta-time exists and, therefore, I continue to hold the position that within the shortest interval of time, it is meaningless to talk about passage of time.

Frequently, branching pattern are chosen to be representative illustrations of structure of future. The more distant future one speculates about, the more possible states of the world there appear to be in such illustrations. This is in stark contrast to the 'plane' that has been developed in this paper which makes no such distinction between near and distant future. An immediate response to resolve this could be the following. We take all the infinite possible world states that would be generated as unique branches in the structure of time in the infinitely distant future and then use that set of possible world states to constitute the $\hat{\omega}$ axis. This is consistent with consideration mentioned earlier i.e. a priori elimination of possibilities is only justified in the formal structure if it stems from an axiom.

The aforementioned response does not resolve the tension. The infinity involved in the branching structure of time is a potential infinity and the infinity involved in our abstract framework is an actual infinity. The branching structure has its characteristic shape because of its assumption that new descriptive propositions are to be generated about the world as time progresses. On the other hand, in our framework, we are considering that all possible world states can be mapped to a set of $\aleph_0$ size. Whether these two infinite sets have equal cardinality needs to verified mathematically and is a non-trivial comparison.

4. **Reach of Causal Influence and Present**

According to Special Relativity, simultaneity breaks down under Lorentz transforms between different frames of references. This means if three events appropriately separated in space seem to occur simultaneously for an observer at rest (for example bursting of three balloons), the three individual events might appear to be happening one after the other for a moving observer. Asking what is happening 'now' is equivalent to asking for the set of events occurring simultaneously with the utterance of the question. Thus, we see the idea of the present moment is integrated with the notion of simultaneity. If distant simultaneity is to be discarded, asking what happens 'now' cannot have a meaningful absolute answer either.

Though this result disallows the existence of a global present applicable to all frames of references, it does not disallow the existence of a phenomenologically distinct present for each frame of reference. If the present moment only reflects the 'parochialism of a particular observer', is there a way to formalize it?

4.1 *Capturing an Intuition*

Human perception appears to have a tensed component that entails an intuitive understanding of the present. Whether it is a result of intrinsic human neurology, universal laws of physics or cultural indoctrination is a difficult open question to answer. However, probing the possibilities to formalize this intuition is still a worthy inquiry. One view that I wish to motivate in this paper is the idea of using the reach of causality in order to prescribe a special status for phenomenologically distinct present (distinct, from past and future).

I assume that agents have the capacity to change world states. The term 'agent' of a change is used in the most general sense to encompass any physical object (which may or may not be sentient) in causal relation with other physical objects. Humans changing the microscopic states of the atoms in the air while exhaling, a virus replicating by rearranging RNA or stars obliterating planets by going supernova, are all examples of agents changing the state of the world. The capacity to create changes is constrained spatially and temporally. This follows from the idea of light-cones in Special Relativity. For every reference frame, there exist horizons beyond which lie events that cannot have any causal relation with the agent; thus establishing the impossibility of the agent to change anything about those events.

In a phenomenological sense too, the span of our capacity to induce a change in the world seems to reside in a special region that we understand as 'now'. Neither can I pluck a flower that will bloom in the spring of year 2121, nor can I pluck a flower that bloomed in the spring of year 1921. Neither can I harm a person who is yet to be born, nor can I save the casualties of WW2. I can only change the state of the world as it is at a phenomenologically distinct present. One can claim that agents do have the ability to change how the world would be in the future. For example, I can feed a hungry bird in the future by growing a mango tree in my garden now. This change, however, can be reformulated in terms of a series of changes of world states. *Now*, I do not feed the hungry bird of the future. Instead, I change the spatial location of the seed of mango tree and displace some soil. In the subsequent moments, the seed absorbs nutrients and undergoes a long complex series of changes; each of which happen in a local bubble that constrains the causal capacity of the agent. Cases which claim to demonstrate an agent's causal relation with the future, simply demonstrate a chain of causal relationships in a series of linked presents. Such examples do point towards a crucial property of the causal bubble of present; it is dynamic. This is compatible with our perception of time that points towards a picture in which time flows. This is the intuition that we shall try to motivate a formalism for, using the framework developed earlier.

Though we have a distinct present that flows, it is interesting to note that this intuition alone does not put any constraint on which direction the present should flow towards. The argument for distinct present only entails that the reach of causal capacity is local. It has no constraints on the direction of the causal capacity. If an experiment was to demonstrate that a type of force affects configurations of a system in - what humans perceive to be - the past, the notion of a phenomenologically distinct present would still hold for that force. Similar to how the notion of causally affecting the state of the world in future was reformulated in terms of a series of changes in a dynamic present for the agent, from the reference of the past-propagating force, any notion of causally affecting the configuration of a system in the past can be reformulated in terms of a series of changes in a dynamic present as well.

4.2 *Formalizing the Intuition*

Recall in the framework developed in last chapter, we had countably infinite world state numbers on one axis and a discretized axis of time as well. Since the axes are not continuous, we can visualize the entire space, not as a plane but as a grid. Each node on the grid represents a possible state that the world could be in at a given instance of time.

Consider an agent that perceives the world to be in some state ω_1. Due to the nature of mapping from world state numbers to time instances, we can unambiguously infer that the agent is located on a node in the grid which has coordinates (t_1, ω_1). Following from the phenomenological intuition discussed in the previous subsection, the agent has a local causal reach and can only influence how the world is at that node. In other words, the agent does not stand in a direct causal relation with with any other world state apart from ω_1 and any changes that can possibly happen due to the actions of the agent, must happen to the truth values of the propositions constituting ω_1.

However, once any change is enacted, the world does not remain in state ω_1 anymore. The world snaps into some other position on the space and the agent's causal reach resides at a different junction on the grid, say ω_2. We also know that changes are accompanied by the increases in global entropy. Therefore, a different clock event proposition now holds true which corresponds to a different value on the $\hat{t}$ axis. Thus, the agent's location on the space is now (t_2, ω_2). The new location of the agent on the space is different from the previous one, not just quantitatively - representing a new world configuration - but also qualitatively because the agent perceives this new location to be phenomenologically distinct in terms of its causal capacity.

The method that I wish to motivate in order to model the intuition formally utilizes the notion of an *operator*. An operator simply takes in a specific object and yields another object. For example, the square root operator takes in a value of 49 and returns the value of 7. The definition of an operator entails a well defined set of values over which it is allowed to *operate* on and generate

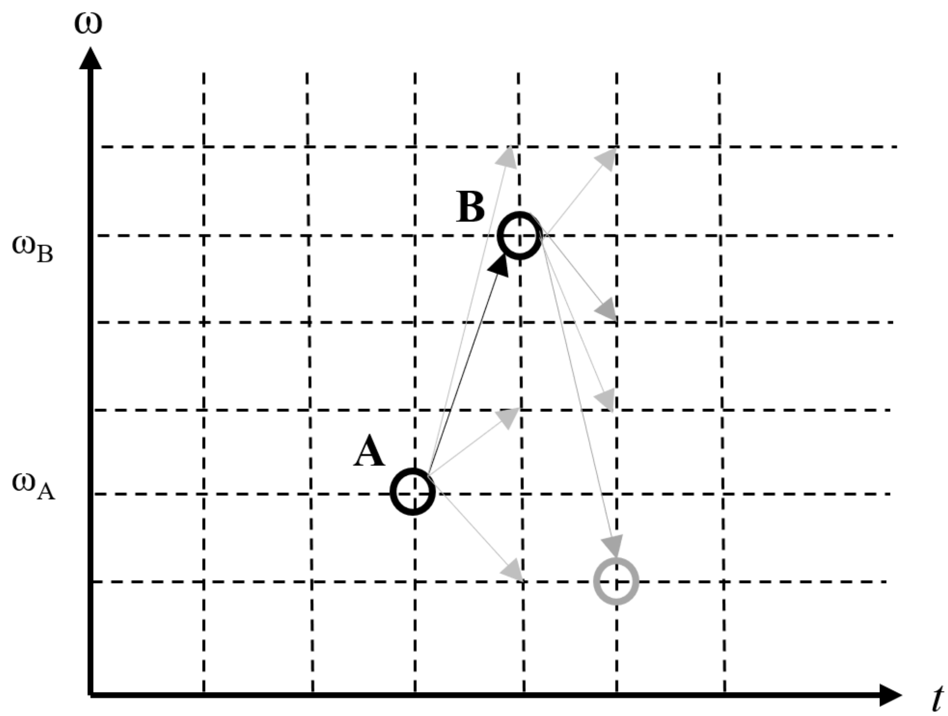

Figure 3: If the agent perceives the world to be in state A, the only changes that the agent can possibly cause are in the truth values of the propositions that comprise ω_A. However, as soon as any change has been caused the world snaps into a different position on the state space (say B) and the agent now perceives its present state B to be phenomenologically distinct from state A as its capacity to cause change is now constrained to truth values constituting ω_B. The agent's causal capacity does not span over junctions on which it does not reside.

meaningful results (like how the square-root operator cannot operate on negative numbers).

The operator that concerns us is the *Change Operator* (denoted by $\mathbf{C}$) that takes in an n-tuple of the form (t_i, ω_i) and returns another n-tuple of the same form. A discussion on the precise definition of the operator is presented in the following chapter but the parts relevant to us now are following -

$$\mathbf{C}(\bar{X}) = \begin{cases} \bar{X}' & \text{if the agent perceives the world to be in } \bar{X} \\ \text{Undefined} & \text{otherwise} \end{cases}$$

Since the domain of the operator is comprised of the set of values that it can operate on while generating meaningful results, the domain of $\mathbf{C}$-operator is strictly only the node on which the agent resides. However, as soon as the operator operates on that junction, the position of the agent changes and so does the domain of $\mathbf{C}$. As would be obvious by now, the dynamic and ever-changing $dom(\mathbf{C})$ is the phenomenologically distinct present that the agents perceive as the flowing time.

The present, in our space, is defined unambiguously as the exclusive region in the framework where an agent's capacity to create changes lies.

> The notion of present being defined as the domain of the change operator naturally leads to an important philosophical question, one which I think is worthy of making explicit here. There is an implicit assumption about the nature of time in the aforementioned formalism. If there is no change, the present does not 'flow'. This is at least internally consistent with the idea that in intervals smaller than that of Planck length, not even entropy can rise (since that, too, is a form of change) and each instance of time is marked with a unique clock event proposition. Thus, for all the duration when absolutely nothing about the world has changed, the world remains in the 'present'.

5. **Philosophical Speculations**

5.1 *Deterministic Time v/s Stochastic Time*

There are deterministic views of time which function under the idea that the history and the future of world at already determined and, if one could know the state of the universe with absolute precision and everything about the way the universe evolves, the world state can be predicted at every instance of time. On the other hand, schools of philosophers who consider that the future is open to revisions and choice (in some 'true' sense of the word) exists, subscribe to some form of stochastic time. There are dozens of diverging ontological and epistemic considerations about the reality and computability of past and future within these two broad categories. However, the phenomenologically distinct present that was motivated in this paper is compatible with both of these views.

As a determinist, one would argue that the **C**-operator is intrinsically algorithmic in nature. By knowing the input n-tuple on which the operator acts, it is (at least in theory) possible to know, hundred percent of the times, the output n-tuple with complete accuracy. As someone who subscribes to a stochastic view of future, one would want to argue that **C**-operator is intrinsically uncomputable and is influenced by countless random fluctuations which are - even in theory - impossible to predict with hundred percent accuracy (for example, decisions made by conscious agents possessing free will influencing the future in a model where consciousness is uncomputable). In either cases, the operator would still have a defined domain over which it can meaningfully operate on. The claim about the dynamic $dom(\mathbf{C})$ representing the 'flow' of time is independent of the internal mechanisms of the operator.

It is also interesting to note that this kind of formalism is compatible with various temporal logics

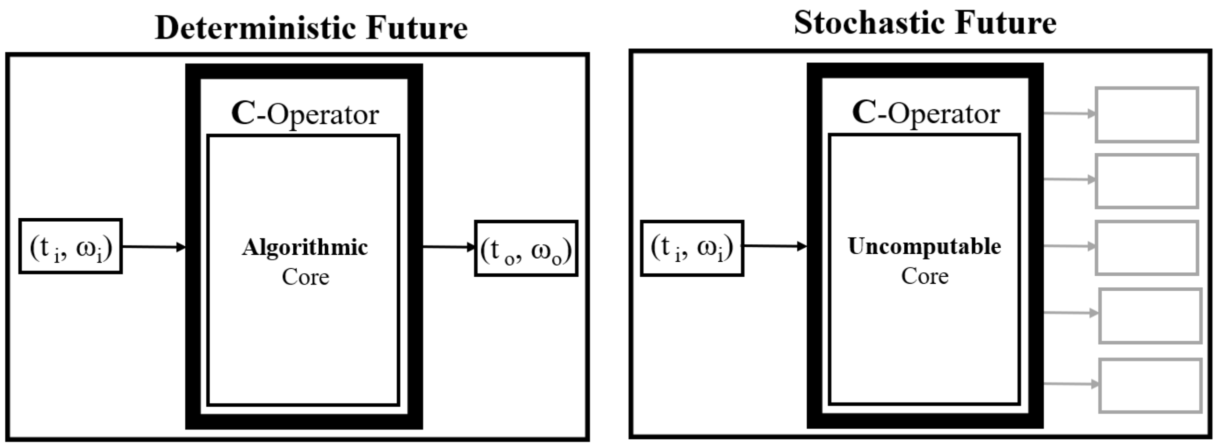

Figure 4: A schematic demonstration of how the formalism for distinct present holds regardless of whether the core of **C**-operator is intrinsically algorithmic or uncomputable. For deterministic futures the output can be generated mechanically and for stochastic futures it can not, but it does not affect the fact that the operator only operates on certain inputs meaningfully.

of special relativity. To embed the aforementioned idea of 'present' in frameworks developed by A. A. Robb, R. Goldblatt, R. Hirsch and B. McLean, and others, one needs to first introduce the expressive power of Operators in the system. Once that is done, a space of possible world states can be generated regardless of the precedence relation in the system (which generally is based on the metric of the line element in Minkowski space).

5.2 *Programming the Emergent 'Flow'*

An elementary python script was developed to model the aforementioned formalism and the code was run for both stochastic and deterministic evolution. The computer was told what the change operator is and an initial world state was provided. In the case of stochastic evolution, the program checked if the current position of the agent is at the node on which the operator is supposed to act on. If the position and input matched, it evolved the world state randomly with two constraints to meet the clock-event postulate - the future state should have never occurred in the past and the global entropy must be higher than that in the input world-state. From the output n-tuple, a time instance was extracted and since the change entailed an increase in entropy, the value of time always increased. Thus, from an initial state that matched the position of the agent on the space, the program recursively drifted the agent's location towards the future.

It was observed that once the formalism was programmed, the 'flow' of present emerged from the logic of time automatically. It is not feasible to embed animations into a PDF but a global-snapshot of the animation has been presented in Fig. 5. Note that, phenomenologically, the agent only perceives one dot on the space at a time that represents the constraints on its causal capacity (equivalently, the domain of the **C**-operator). The dot then flows towards right (representing future by convention).

The same was done for a deterministic evolution of time by simply changing the method of evolving world-states in the **C**-operator by replacing the random core of Stochastic Operator with a well defined linear mathematical formula. The drift towards future was noticed in this case as well.

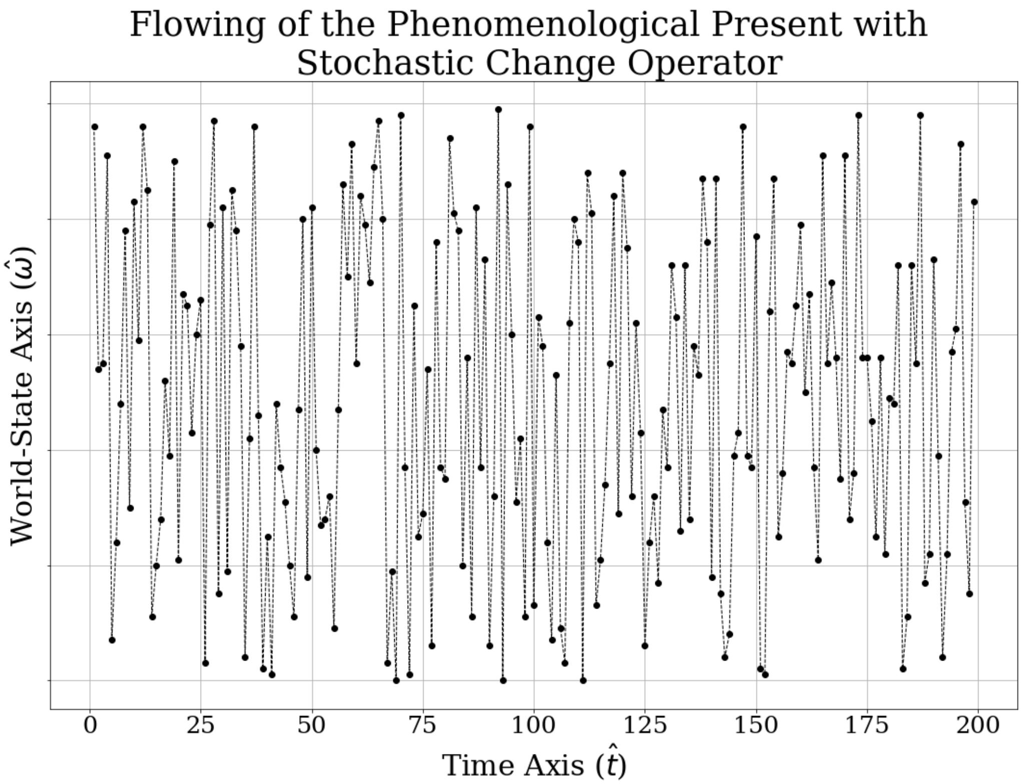

Figure 5: Flowing of the *present* emerging from the intrinsic structure of our phenomenological formalism

6. **Epilogue: A Fatal Flaw (?)**

When this independent study was on the brink of its completion, I noticed a tension between an important theorem in mathematics known as Diagonalization Theorem and the coding to world-state numbers developed in Chapter 2. In this section, I explain the problem, discuss its implications and speculate about possible resolutions. The reader is encouraged to revisit §2.1 to §2.3 for the necessary context.

Assuming the world can be described by an infinitely long binary array made up of truth values of descriptive propositions about the world, I compare the number of possible world states with the set of natural numbers. We start with the traditional method of comparing infinities using the

Diagonal argument and start listing all world state strings in-front of all natural numbers. Then, we start to form the diagonal string by highlighting first digit of the first string, second digit of the second string, third digit of the third string and so on.

1 : 101000101010010....
2 : 101001010100010....
3 : 010001010001010....
4 : 010011111010101....
.
.
.

We define the rule that for each digit in the diagonal, if we encounter 0 we change it to 1 and vice versa. Thus, we have a diagonal string $\mathbf{D} = 0111...$ that differs from any N^{th} natural number *at least* at the N^{th} digit. This has all been well establish and sound.

My tension arises when I start discussing the coding mechanism to map the binary strings to natural numbers using prime factorization. The way I perform this coding is by using the value of the N^{th} digit of the infinite string and raising the N^{th} prime number by that value. For example, using the diagonal string,

$$\mathbf{Z} = 2^0.3^1.5^1.7^1...$$

This is a completely unambiguous mapping which can be performed for any given binary string. Further, by multiplying natural numbers, we cannot get anything other than a natural number (or so I thought!). This establishes that all possible world states (which are described by the infinitely long binary strings) can be mapped on to natural numbers. But, the diagonal argument demonstrates that the two infinities are unequal. Even if there would always be a diagonal string that would never fit in the infinite set of natural numbers, there would also always be an isomorphic natural number corresponding to it which would exist in the set of natural numbers.

A helpful discussion with Prof. Sharon Berry brought to my attention the potential source of this inconsistency. A counter intuitive result in mathematics establishes that the product of an infinite set of natural numbers does not yield a natural number. Therefore, a binary string with infinite 1s cannot be mapped to a natural number because it would entail multiplying infinite primes together, yielding a number that is not a member of the $\mathbb{N}$.

At the moment, rejection of using a set of size $\aleph_0$ for representing World States also requires rejection of discretization of world state axis. A non-discrete world-state axis impacts a part of the argument that calls for discretization of time and certain changes in vocabulary are required in sections discussing the snapping of agent's position to different junctions on the World State space.

The discussed coding mechanism is definitely flawed and needs to be discarded for the infinite cases. It is my opinion that the essence of the argument that utilizes the domain of the change operator to represent the agent's phenomenological causal bubble flowing through time can still be preserved. One method to do so might be using a finitely long binary array of propositions as an approximation of the real world. Another way to go about it would be to employ a different strategy of representing world-states on a space. A resolution of this problem (or a method of circumventing this problem) is left as an open question to the community.

Even though some sections of this project have now been proven to be invalid and require revisions, I hope other sections contribute some value to the field of temporal logic. Further, for logicians working in temporal models with enough expressive power to include the notion of operators, the aforementioned discussion may provide practical utility with conceptual gaps that need to be covered in future work.

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