Kartik Tiwari

Prof. Scott Dixon

Philosophy of Science

2 June 2021

Incompatibility of Subjectivism and Convergence in Bayesian Thought

Within the context of Bayesianism in Philosophy of Science, subjectivist interpretation of probabilities and convergence leads to a confusion. I try to demonstrate how that confusion arises due to an incompatibility and attempt towards a clarification and possible resolution. First I shall briefly introduce the two incompatible ideas (convergence and subjectivism) and present their equivalent formulations to streamline my argument. Then, I will demonstrate the incompatibility of the two concepts and, finally, discuss the road ahead.

According to the subjectivist interpretation, probabilities can be understood as degrees of confidence. By stating that the probability of an event X is P(X), one is essentially claiming that they have P(X) confidence that event X would occur. The same idea follows in understanding claims and degrees of credence. One does not simply believe a claim to be true or false but has a certain degree of confidence on the truthfulness of the claim. In other words, probability is not something external to the subject and inherent to an event or claim but is rather characterized by the relationship between the subject and the event. Thus, it makes little sense to simply state P(X) to be a certain number. Rather, the complete description of probability would be $P_i(X)$; which a subjectivist would read as 'Person i having $P_i(X)$ confidence in the claim X'. The important takeaway here is that a complete subjectivist description of probability requires at least two parameters, i and X.

Moving on, a mathematically well-probed property of Bayesian probabilities and statistics is *convergence.* It is used to circumvent the Problem of Priors by demonstrating that irrespective of what initial probability one assigns to an event, after enough successive iterations the probability tends to converge towards a specific value. If we start with a sample set of people M each having a wildly different degree of confidence assigned to a hypothesis h before being exposed to n pieces of evidence then the law of convergence states that for all $i \in M$, $\lim_{n \to \infty} P_i(h|e_n) = k$ (k being some constant between 0 and 1). One could get rid of the infinity by reformulating convergence as an in-the-limit property. However, the core content of convergence remains that every person in the sample (regardless of their initial confidence in h) reaches infinitesimally close to a common degree of confidence when presented with a large enough number of evidence. Since this statement is completely general, it applies to any set of people with any distribution of prior confidence on hypothesis h.

Neither convergence nor subjectivism comes without philosophical criticism about their validity but the purpose of this paper is not to argue against either of them. It is simply to demonstrate their incompatibility with each other. To do so, we start with a large set of isolated subjects with a random distribution of prior confidence on h. There is no mechanism for the subjects to interact with each other which ensures the impossibility of having any causal influence on other's confidence. Each person in the sample assigns h a probability which, under the subjectivist interpretation, is a relationship between the subject and h as characterized by the degree of confidence one has in the claim and not something inherent to the claim itself. However, due to convergence, after a large number of Bayesian updates, regardless of which individual you pick from the sample, he or she would have the same degree of confidence on h. The choice of the person in

the sample becomes insignificant to the probability of the claim as the priors wash away with each update. Therefore, at this stage, one can provide a complete description of the probability of h being true without any reference to the individual people concerned.

The above situation goes against the subjectivist interpretation in at least two ways. Firstly, the problem is that subjectivist interpretation requires probabilities to be characterized at least by two parameters (the claim and the individual concerned) but we arrived at a complete description of probability without any reference to the individual. This is a relatively weak attack because a subjectivist can counter by saying that even though we are not explicitly referencing the individual when discussing the probability after convergence, the parameter is still implicitly embedded within the concept of probability itself. This means, we would still interpret the probability as the degree of confidence a person has in the claim but in the special case we have 'a person' can be substituted with any person from the sample. Therefore, what we have is simply a special case of probabilities; nothing that cannot be explained by subjectivism.

The second problem arises when one tries to trace where is the probability originating from. Under the subjectivist interpretation, it is rooted in the relationship between the individual and the event. However, convergence clearly demonstrates that if this relationship exists then it evolves towards something. The effect of an individual's bias, education, culture, etc. on his or her confidence over the claim keeps diminishing as the probability approaches a specific constant value. After an infinite number of updates (or its equivalent in-the-limit formulation), the effect of that individual's personal views over the claim reduces to none and we are left with a probability purely adopted by considering the evidence surrounding the claim. One way to think about it is that there exists a kernel which represents the true probability of a claim which gets obscured and convoluted by the

presence of the fog of personal subjective bias. Each evidence presented to the individual blows away the fog to some extent and brings the probability closer to the true one. Obviously, this must mean that there exists an objectively correct probability associated with the claim which is characterized by the claim itself and nothing else. Without this interpretation of probability, it gets extremely difficult to explain how any arbitrary large number of individuals with their unique subjective experience of the world - without any causal influence over each other - mysteriously approach the same number in a manner as if it was pre-coordinated.

The second problem is harder to navigate and forms the crux of the aforementioned incompatibility. Fortunately, its clarification is a straightforward three-sentence project. There exist schools of Bayesian thought which are objectivists in their interpretation of probabilities. Only objectivists use convergence to solve the problem of priors. Whereas, if you are a subjectivist, then the utility of convergence remains unavailable to you. To prevent inconsistencies in logic, you must choose between the two. To be fair to Godfrey-Smith, he does mention '*Majority* of philosophers who want to use Baye's Theorem ... hold a subjectivist view of probability'. However, an explicit discussion of a formulation of Baye's ideas with an objectivist interpretation of probabilities would have prevented this confusion.

The attempted clarification, however, makes no effort to actually resolve the incompatibility. Since both concepts are important cornerstones of Bayesian thought, there rests some value in trying to find a method to ensure subjectivism can coexist with convergence. Towards the end of my paper, I wish to motivate a possible solution in the form a mild-subjectivism (or, equivalently, mild-objectivism). One way to resolve the incompatibility is if subjectivists grant the existence of an objectively true probability

associated with the claim. Would it not go against a foundational principle of subjectivist interpretation of probability? I argue that after some minor adjustments, it need not.

There is an epistemological concern associated with the idea of the 'true' probability of a claim's truthfulness. How does one know the true probability if convergence only asymptotically approaches towards it? Objectivist Bayesians would argue that even if it is impossible to know the true probability, for all practical purposes we can use limit formulations and get as close to the true probability as the situation demands after collecting enough evidence. However, a hybrid-subjectivist (who grants the existence of 'true' probability) may approach this question by admitting that it is impossible to know the 'true' probability and, therefore, for all practical purposes we deal with probabilities as interpreted by subjectivism. The idea of a hybrid subjectivism (or moderate subjectivism) was briefly hinted in the Godfrey-Smith reading but not explicated upon. Using the hybrid-subjectivist stance, subjectivism can appeal to convergence for bypassing the problem of priors without having to give up on the idea of using probabilities to characterize the relationship between a subject's degree of confidence and the claim.

I started this paper by trying to demonstrate how subjectivism can turn out to be incompatible with convergence because convergence requires the existence of a 'true' value of probability inherent to the claim not affected by the subject's association, which is contradictory to a foundational idea in subjectivism. A trivial resolution of the incompatibility would have been to claim 'Subjectivism is wrong' or 'Convergence is wrong'; there are enough philosophical critics of both. Since both the concepts lend useful results in the theory of evidence via Bayesian understanding and because the purpose of this paper was not to disprove either of the two, towards the end I attempted a potential resolution. To do so, I presented the possibility of resolving this incompatibility by tweaking

traditional subjectivism to create a hybrid (more moderate) form of subjectivism that retains the capacity to use convergence and the utility it provides. There is still scope for further scholarship to gauge the impact that hybrid subjectivism might have in the larger context of Bayesianism and the associated epistemological issues.

Works Cited

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