On Infinities of Spacetime

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> *Now unconfined the wings stretch out to heaven,*
> *Nor shrink beneath a crystal firmament*
> *Aloft into the aether's fragrant deeps,*
> *Leaving below the earth-world with its pain,*
> *And all the passions of mortality*
>
>   - Giordano Bruno,
>   *On the Infinite Universe and Worlds*

The *structure of infinity* plays an essential role in the study of spacetime models. In general relativity (GR), radiation, energy, and isolation are characterized using boundary constructions that carry conceptual commitments often left implicit. I argue that *asymptotic symmetries* provide a useful lens for examining these commitments, connecting philosophy of spacetime with ongoing theoretical and empirical research in physics and cosmology.

1 Infinity and General Relativity

Infinity enters scientific theories in several ways (Fletcher et al., 2019). First, it appears in infinitesimals and limits, philosophically contested since the time of Berkeley (1735) and later treated through successive formalization (Arkeryd, 2005). In GR, limits prompted new discussions about how a field on one manifold could 'approach' a field on another manifold (Geroch, 1977).

Famously, infinities appear in pathological singularities, often interpreted as an inadequacy in a theory's explanatory prowess. Since the singularity theorems show that such breakdowns follow from physically reasonable assumptions, they have rightly attracted sustained philosophical attention (Curiel et al., 2025).

A third, comparatively under-examined infinity represents 'metrically infinitely distant' spacetime boundaries. This notion features in the study of asymptotic symmetry groups, conformal completions of spacetime and various low-energy phenomena. Given its promising role in classical, semi-classical and quantum gravity, and its growing empirical relevance, the infinity of asymptotic structure has received insufficient philosophical attention. My doctoral research addresses this gap.

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2 Asymptotic Symmetries

Early relativists debated whether gravitational waves (GWs) were physically plausible or artifacts of linearising a non-linear theory. This ambiguity was mostly clarified by Bondi et al. (1962) using carefully constructed coordinates and series expansions. While analysing this framework, Sachs (1962) identified a striking feature of its symmetry structure. He showed that the symmetry group acting on the boundary of an asymptotically flat spacetime, now known as the Bondi–Metzner–Sachs (BMS) group, is much larger than the symmetry group acting on flat spacetime. Thus, some of the Einstein Field Equations' most relevant solutions, including those representing isolated systems emitting GWs, do not reduce to the empty spacetime of special relativity in the infinitely distant, weak-field limits. Using conformal completions developed by Penrose (1963), Ashtekar and Hansen (1978) clarified the geometric meaning of the BMS structure at null infinity and related it to the spatial infinity of Arnowitt–Deser–Misner formalism (Arnowitt et al., 2008) as studied by Geroch (1972).

Later, Strominger and Zhiboedov (2014) showed that asymptotic symmetries are associated with permanent distortions imprinted by passage of GWs (called gravitational memory) and with low-energy scattering relations (called soft theorems). These links are now understood as instances of a more general 'infrared triangle,' with analogues across several domains including cosmology. Thus, asymptotic symmetries occupy a promising position in contemporary research on GWs, black hole information, and flat-space holography (Strominger, 2018). With LISA forecasts claiming potential empirical access to memory effects (Goncharov et al., 2024), conceptual clarifications regarding asymptotic structure now become increasingly pressing.

3 Philosophical Considerations

3.1 Cosmological Boundaries

A first set of philosophical inquiries concern the mismatch between idealizations made by relativists and cosmologists about the asymptotic structure of spacetime. Asymptotically flat spacetimes model isolated systems embedded in an otherwise empty background. This idealization violates the cosmological principle, according to which the universe is isotropic everywhere on the largest scales. Cosmological models impose mathematically distinct boundary conditions, associated with different symmetry groups, memory effects (Tolish and Wald, 2016) and consistency relations (Horn et al., 2014). A systematic account is, therefore, needed to connect asymptotic flatness of foundational models in GR with the boundary conditions of cosmologically relevant spacetimes.

3.2 Asymptotic Reasoning

The boundary construction of an explicitly 'unphysical' (Ashtekar, 2015) conformal completion of spacetime is a unique case of asymptotic reasoning (Batterman, 2010; Fletcher et al., 2019). Does the name 'unphysical metric' represent a genuine ontological stratification within the formalism, or merely a ubiquitous misnomer? On what basis, if any, can asymptotic reasoning using limits be considered equivalent to asymptotic reasoning with unphysical metrics? These issues also invite a re-examination of the role(s) that boundary conditions play in the explanatory and predictive character of a theory (Curiel, 2016).

3.3 Representational Choices

A third set of questions, well suited to an integrated historical-philosophical approach, concern representational choices and their epistemic warrants. In treating spacetime infinity as a collection of geometrically structured points, numerous formal restrictions are required to derive key theorems (such as positivity of ADM energy). How are these choices shaped by the evolving pragmatic constraints of available mathematical machinery? How well do they reflect independently motivated physical intuitions about infinitely distant spacetime? The technical literature contains many such representational decisions whose epistemic consequences remain largely unexamined.

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