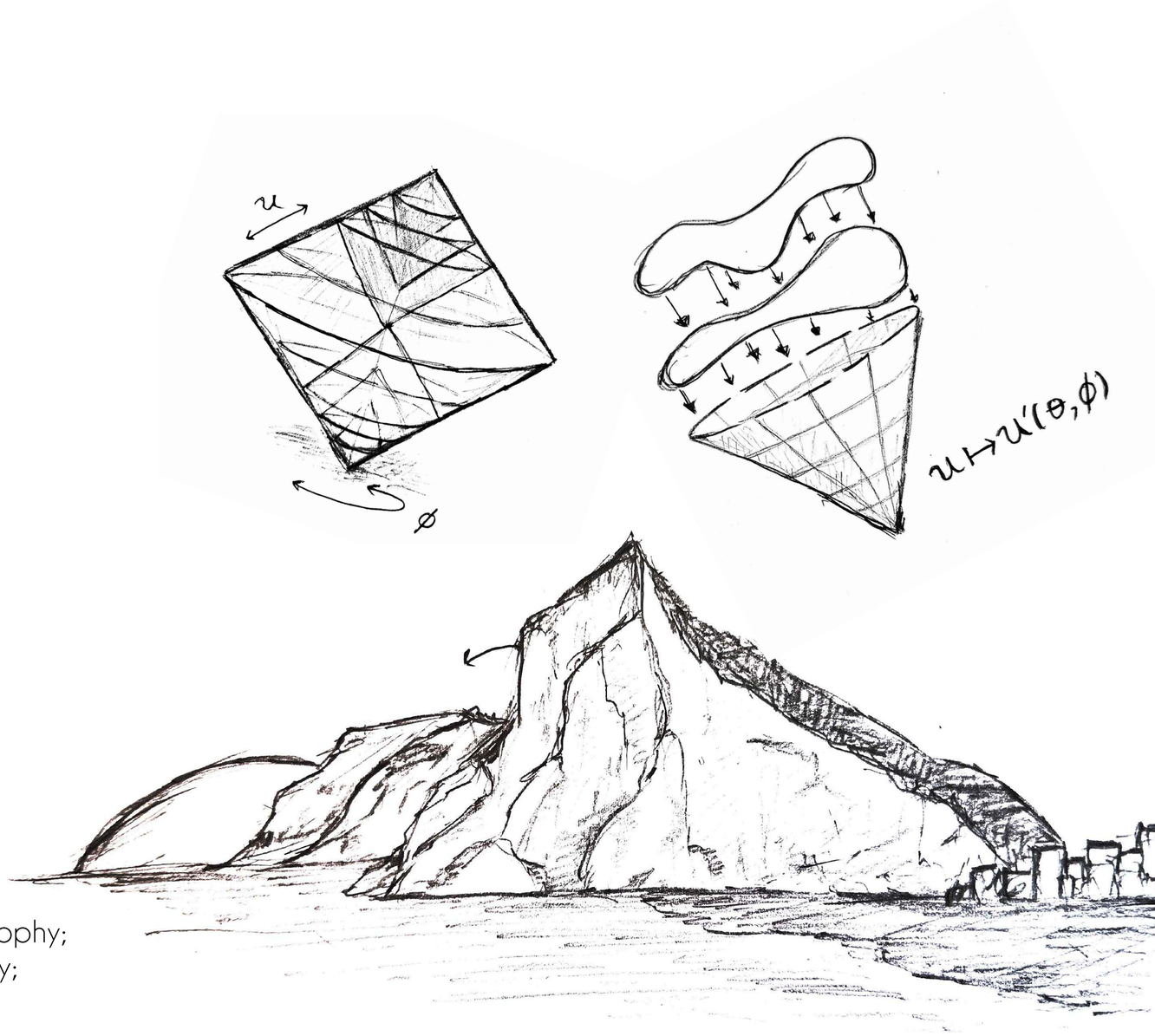
On the Problem of de Sitter Asymptotia



Kartik Tiwari

Lichtenberg Group for History and Philosophy of Physics, Institute for Philosophy;
Large Scale Structure Cosmology Group, Argelander Institute for Astronomy;
University of Bonn (Germany)

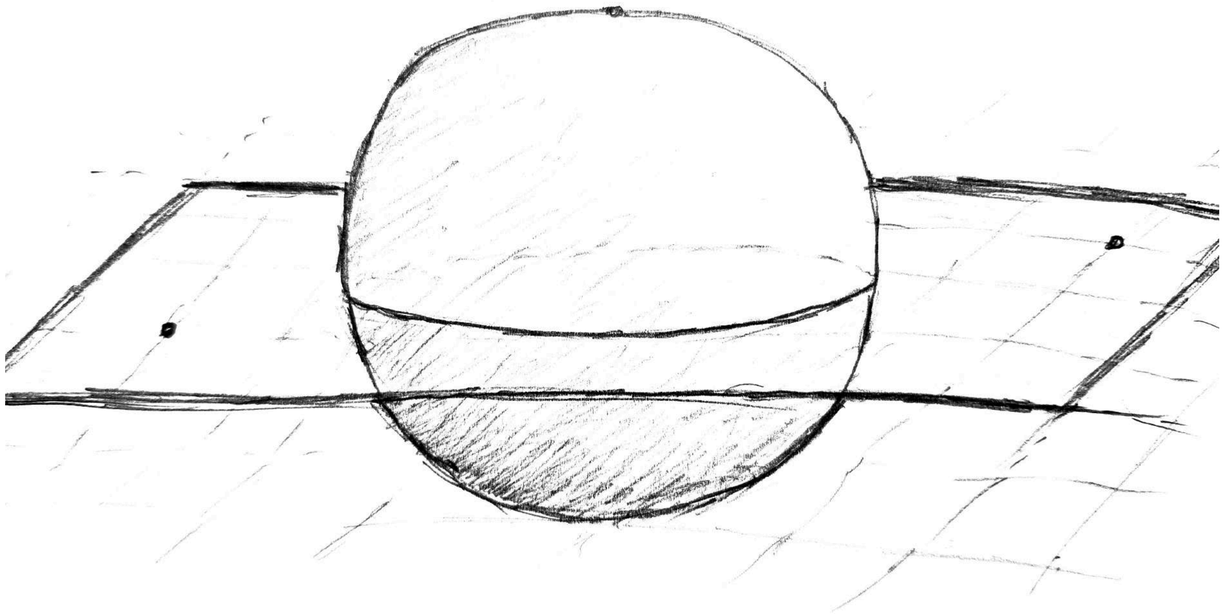
Now unconfined the wings stretch out to heaven,
Nor shrink beneath a crystal firmament
Aloft into the aether's fragrant deeps,
Leaving below the earth-world with its pain,
And all the passions of mortality.

– Giordano Bruno, *Del l'Infinito, Universo et Mundi*
(~450 years ago, during a very grumpy visit to Oxford)

Campo de' Fiori Square in Rome, where Bruno was burnt at stake

Prologue

conformal mapping
of a
boundless complex **plane**
onto a
compact Riemann **sphere**



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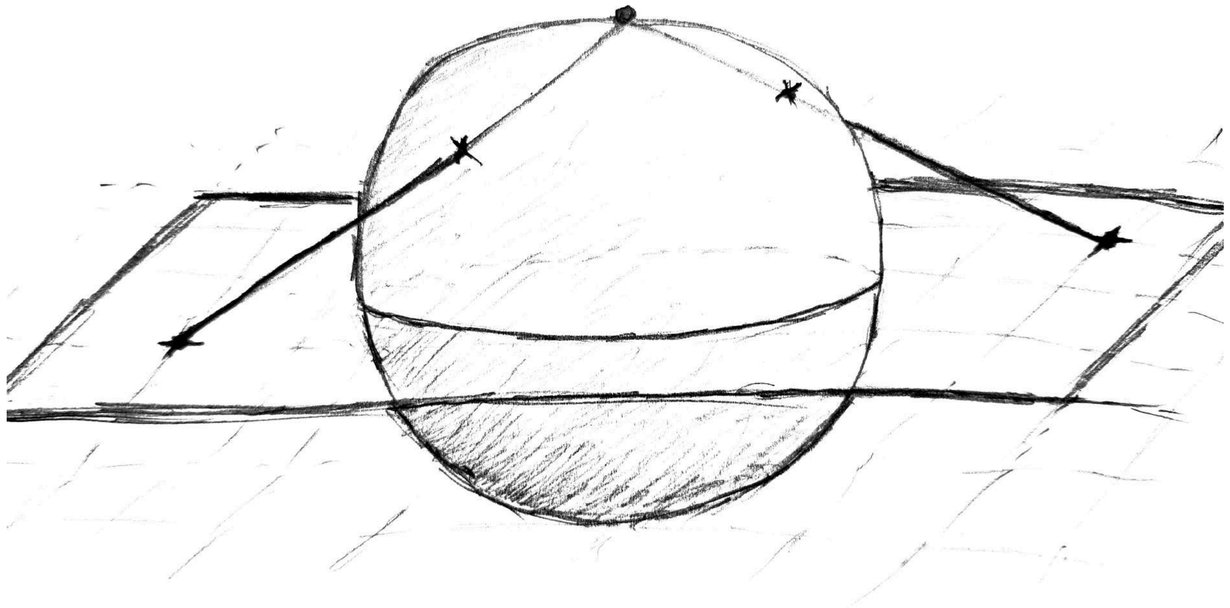

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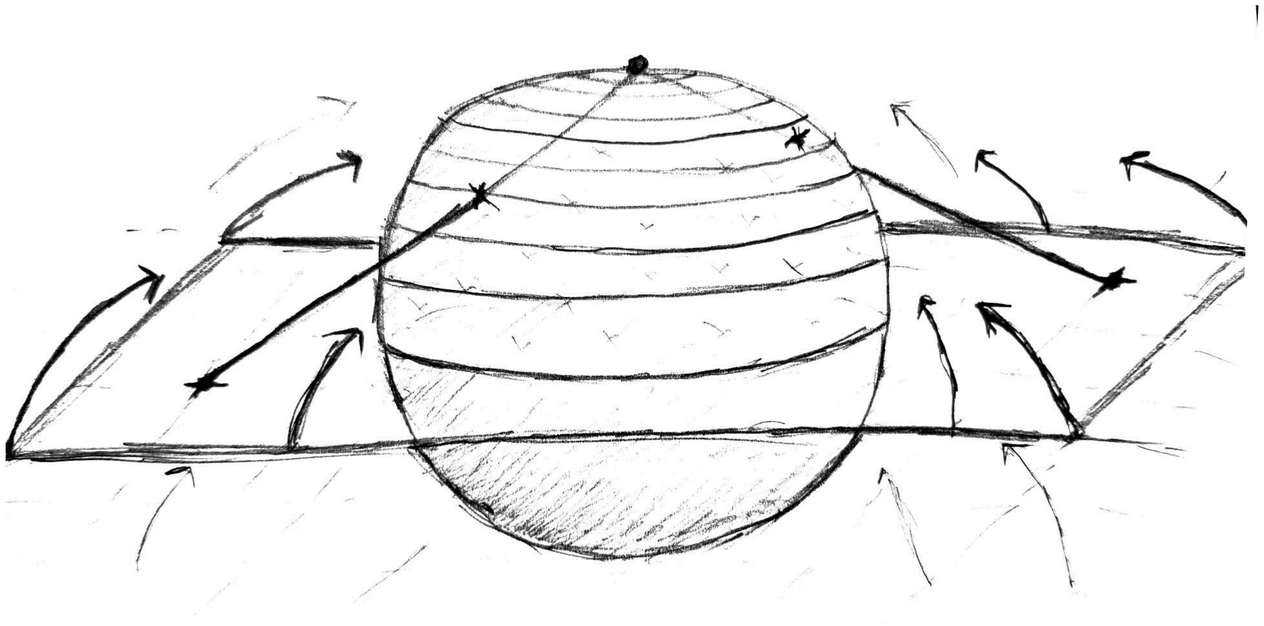

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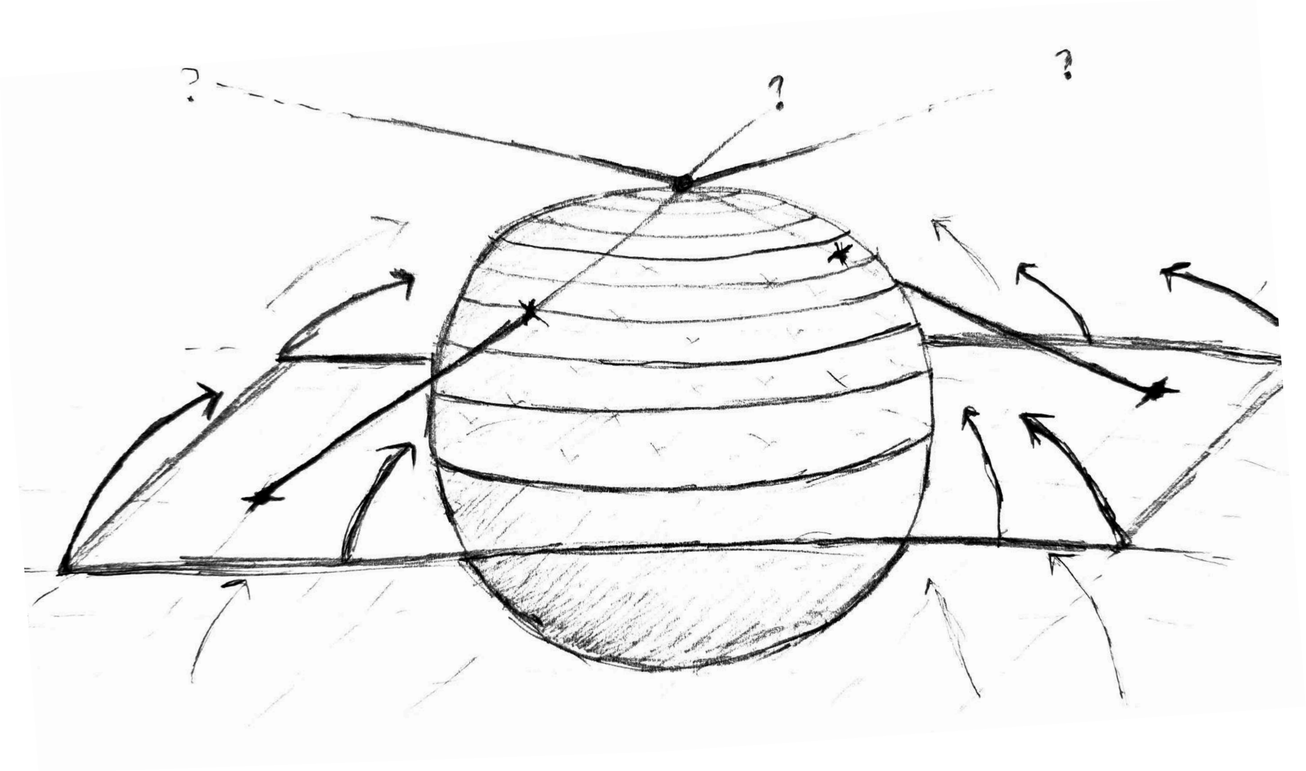
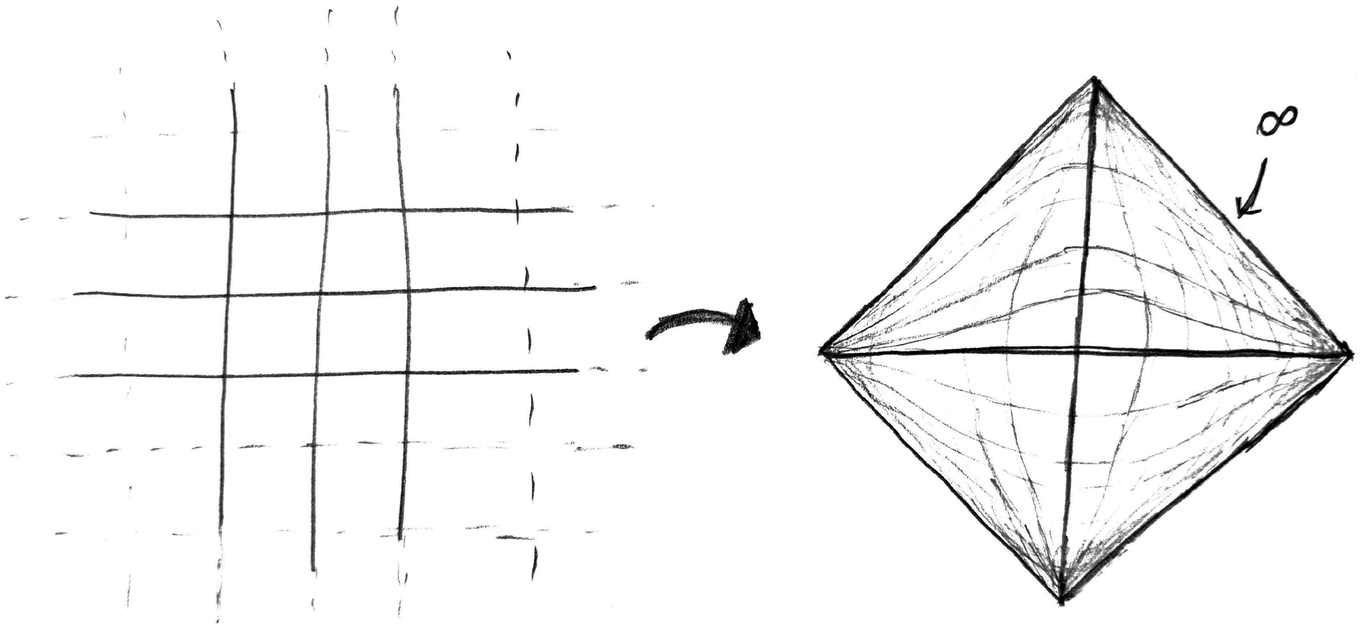
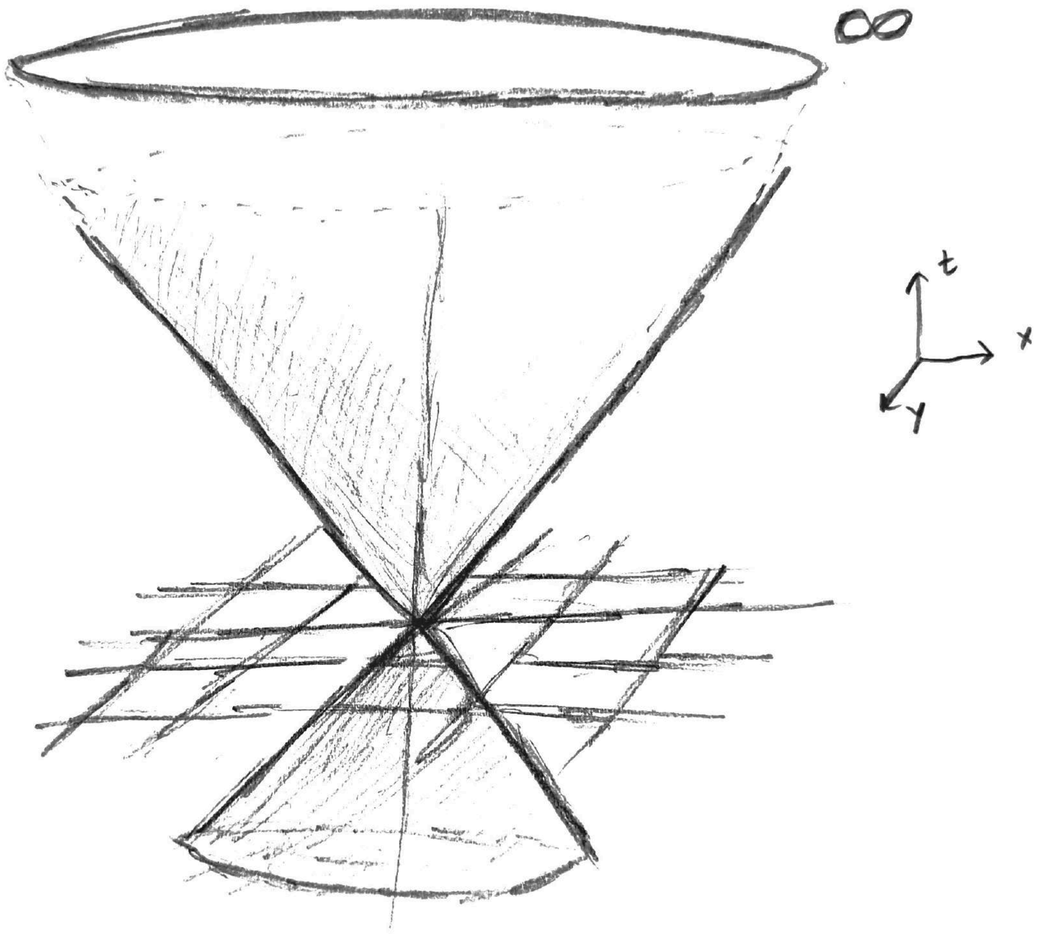


Prologue

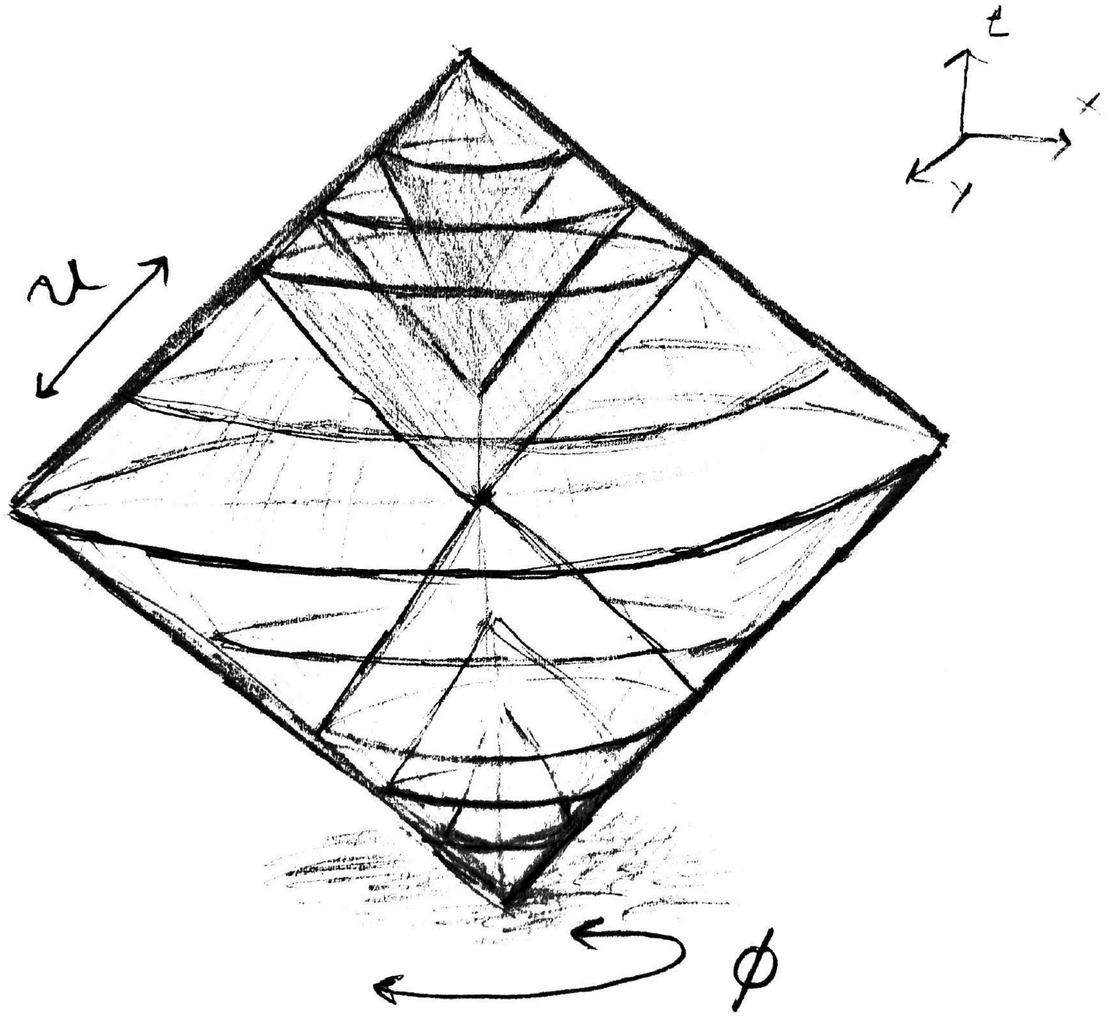
conformal mapping
of a
boundless spacetime manifold
onto a
bounded compact surface





Null Infinity

where **light rays** arrive
having travelled
infinitely far from
an **isolated source**



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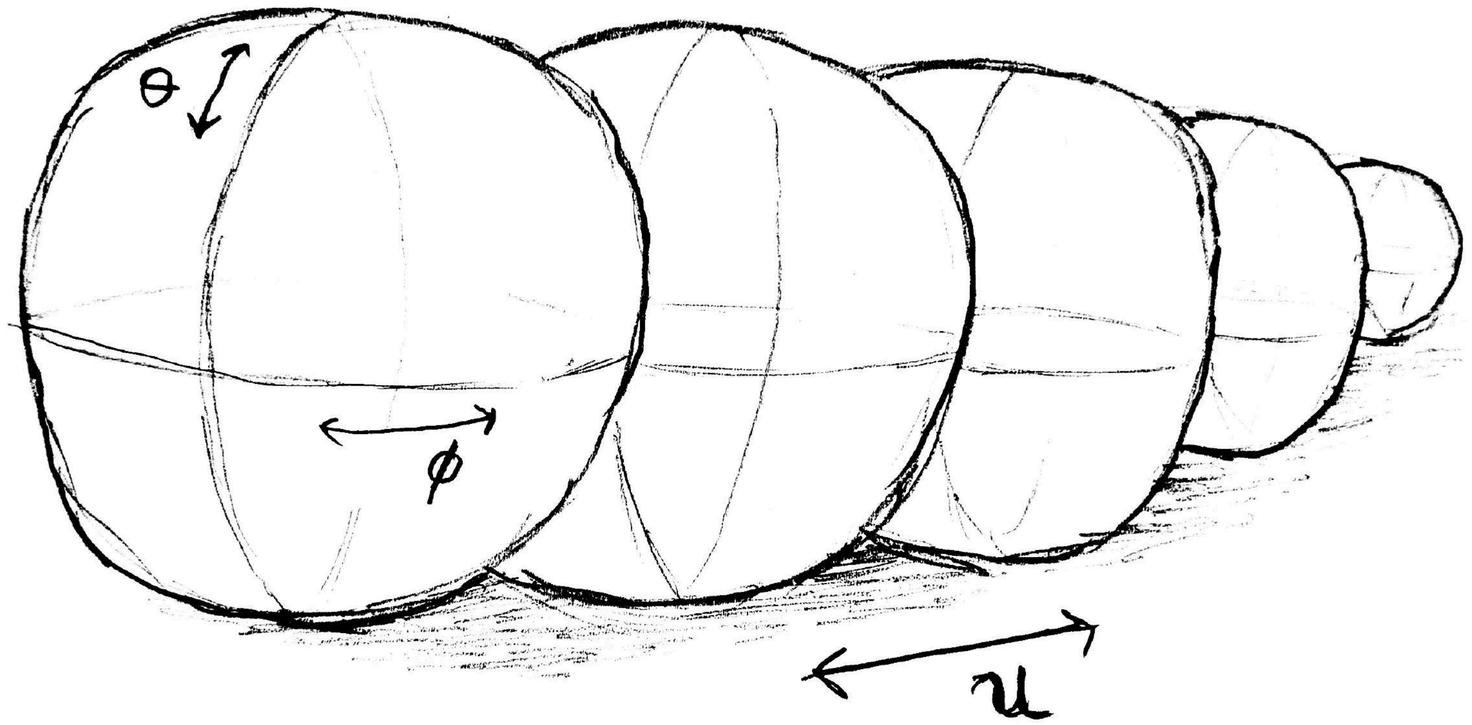

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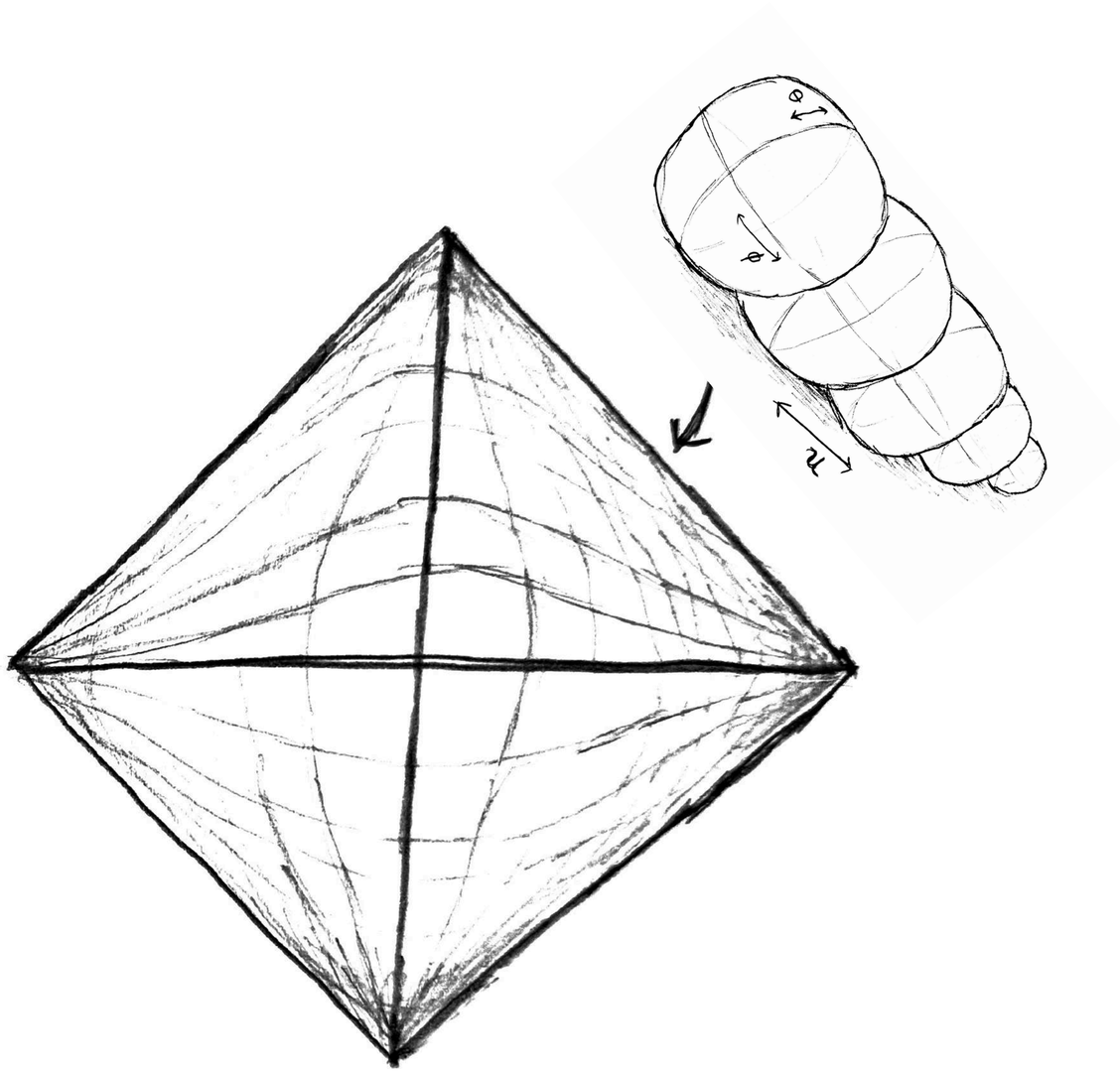

Null Infinity

where **light rays** arrive
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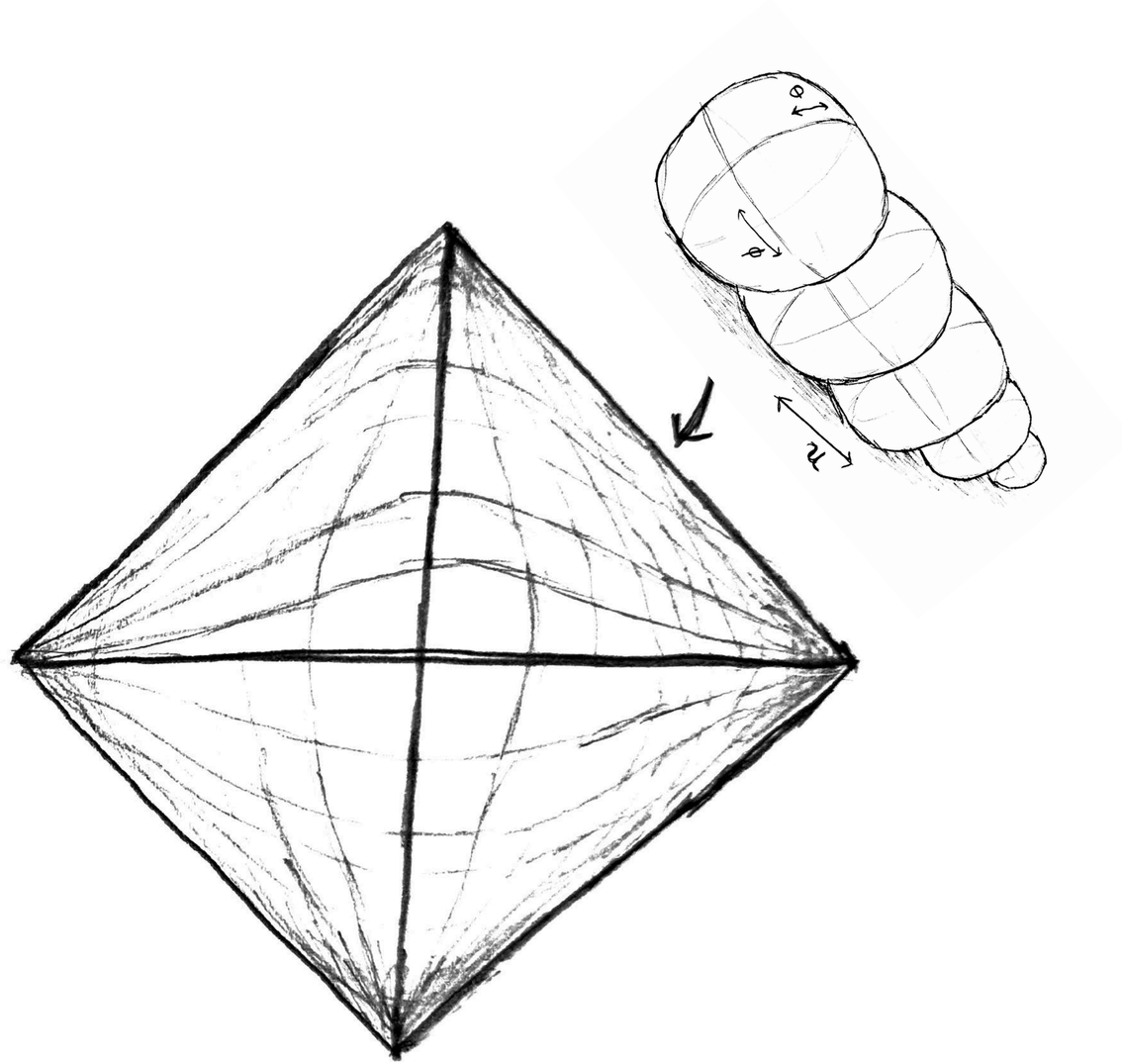
Some important physics is **only** well-defined at this attached boundary (i.e. null infinity)

e.g. gravitational energy, gravitational radiation, gravitational wave memory, etc.



Some (modest) goals

States of affairs infinitely far bear on understanding of local phenomena?!?!

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Positive cosmological constant, no matter how small, revises our understanding of gravitational waves

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Different aspects of null infinity suggest different approaches towards cosmological de-idealisation

Some (modest) goals

States of affairs infinitely far bear on understanding of local phenomena?!?!

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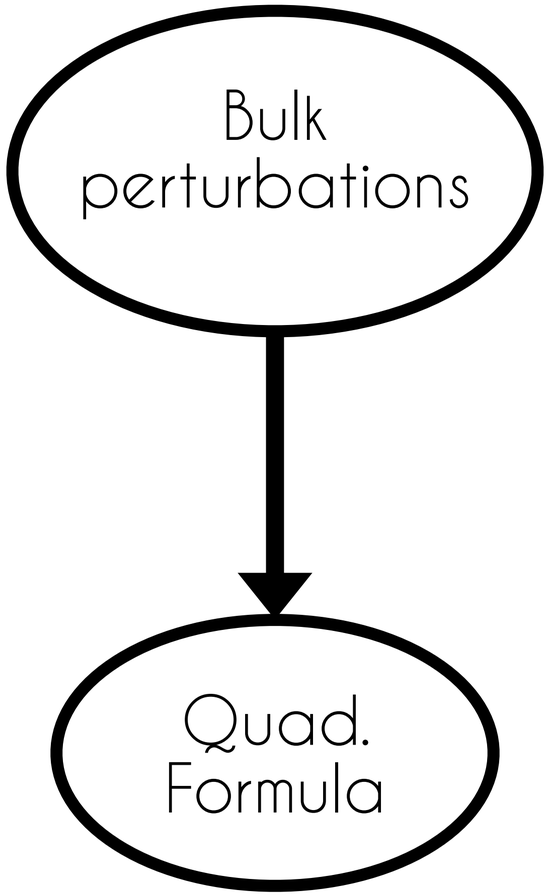
Different aspects of null infinity suggest different approaches towards cosmological de-idealisation

Future observables provide impetus for this de-idealising pursuit

Some (modest) goals

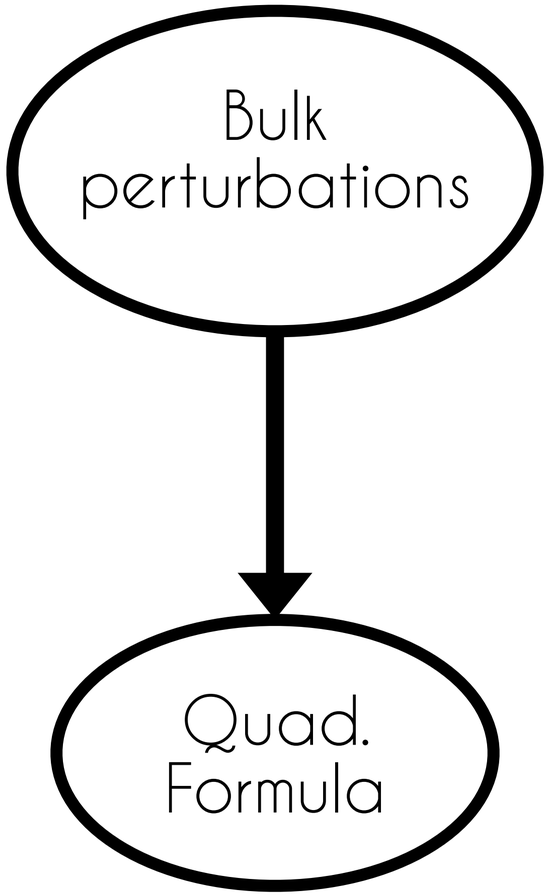
Bulk to Boundary and Back

Why we go to infinity and
how is it used in practice



Very easy to get linearised
GW solutions in specific coordinates

BUT

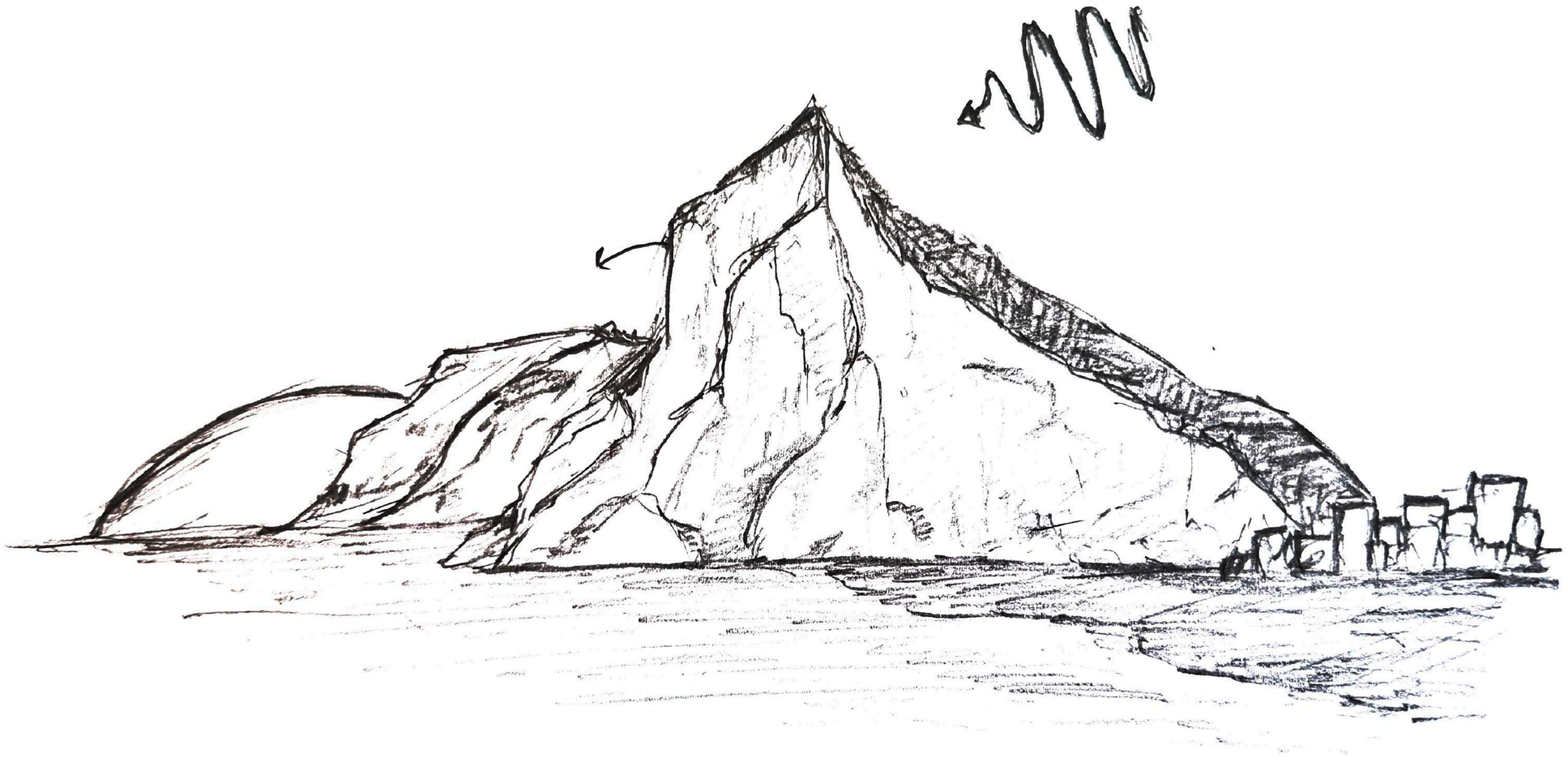


Very easy to get linearised
GW solutions in specific coordinates

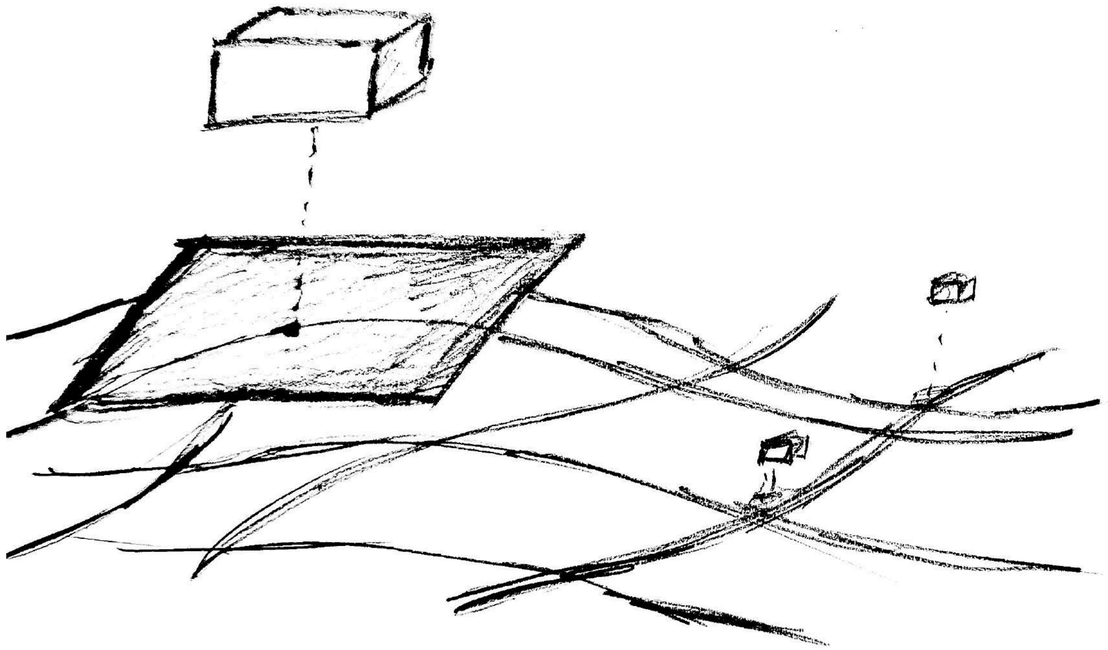
BUT

Coordinate dependent. Hence, **physicality suspect**!

Physicality ~ Carrying Energy

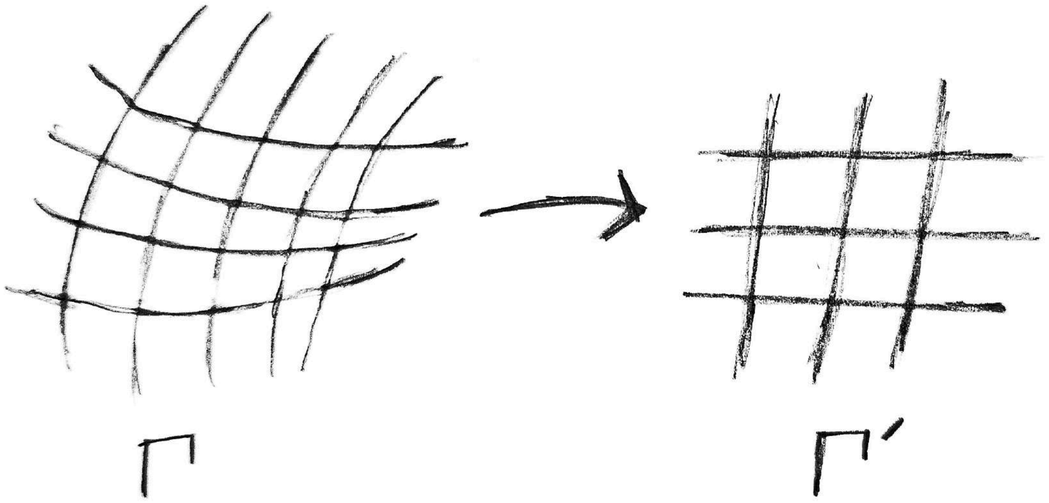


Non-Existence of **Gravitational Energy Tensor** ('folklore')



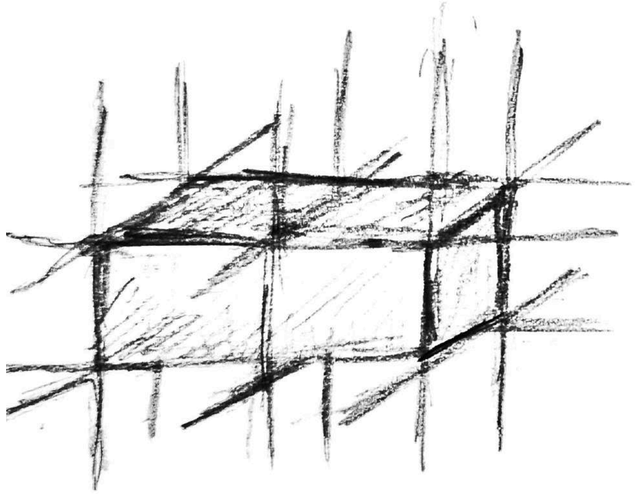
Tensors are **localised** objects that live on tangent spaces

Non-Existence of **Gravitational Energy Tensor** ('folklore')



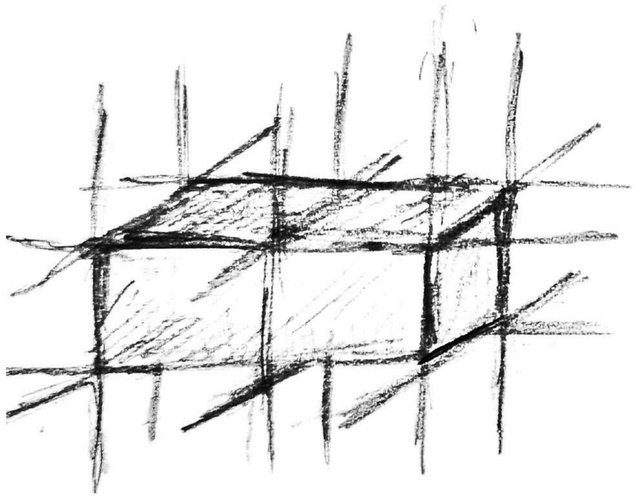
Gravitational field
can always be made
to **vanish locally**
(Principle of
Equivalence)

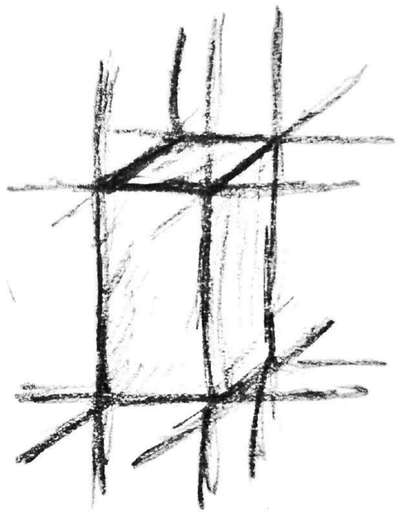
Non-Existence of **Gravitational Energy Tensor** ('folklore')



With scaffolding of ***extra structure***, a localisable gravitational energy can be constructed

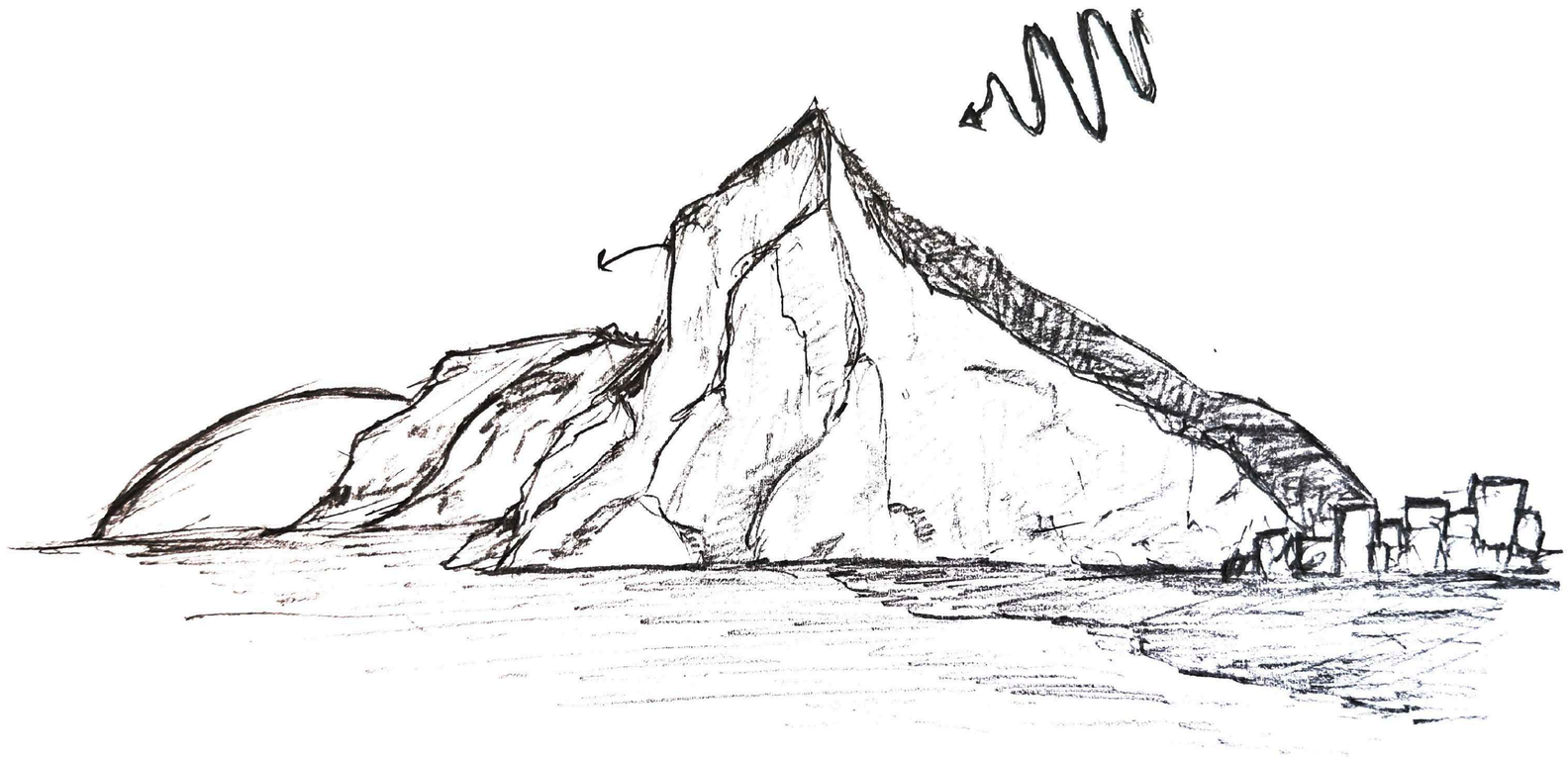
Non-Existence of **Gravitational Energy Tensor** ('folklore')



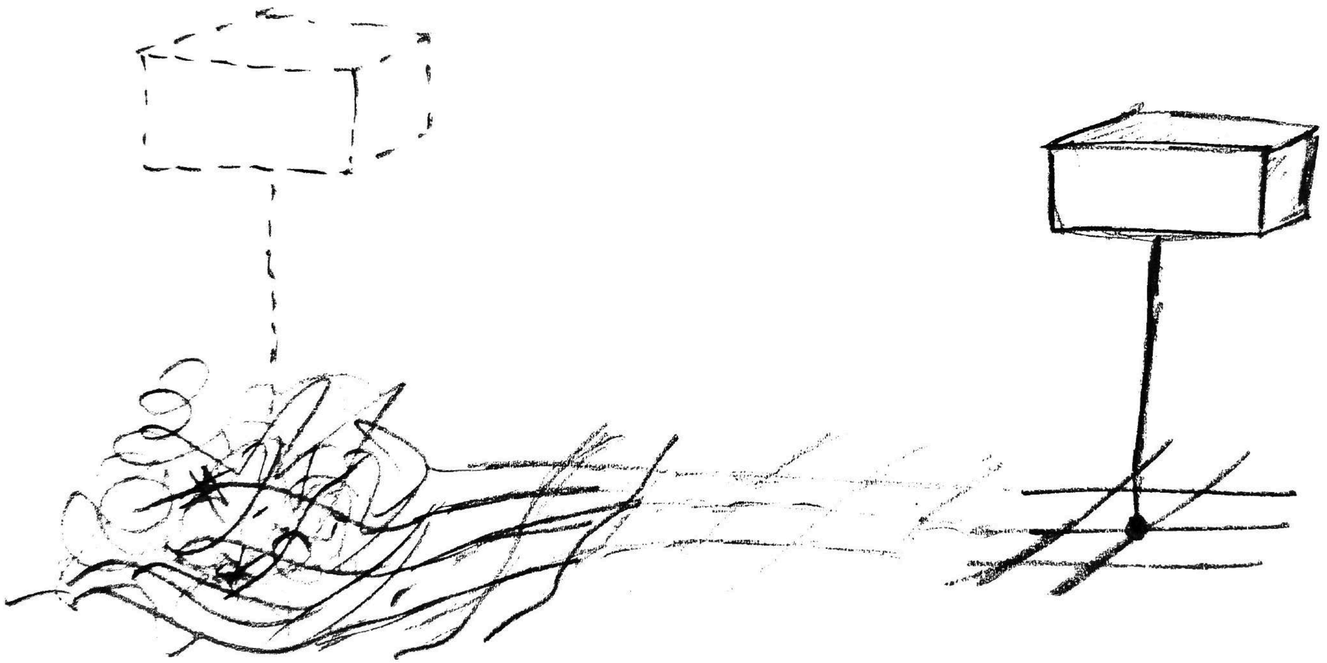


However, **coordinate changes** can **influence the behaviour** of energy flux

Physicality ~ Carrying Energy

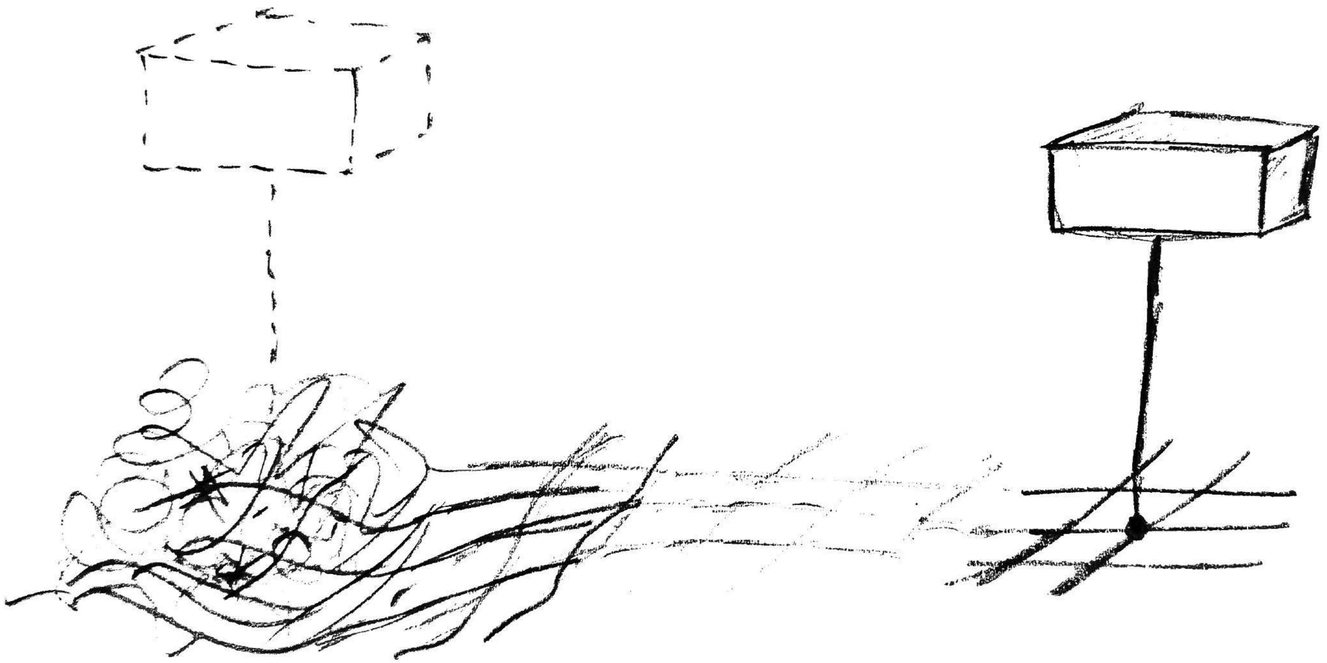


No agreement. Lots of confusion. **Until 1960s**



With asymptotic Flatness,
at **null infinity**,

Radiative modes
decay slower than
static modes

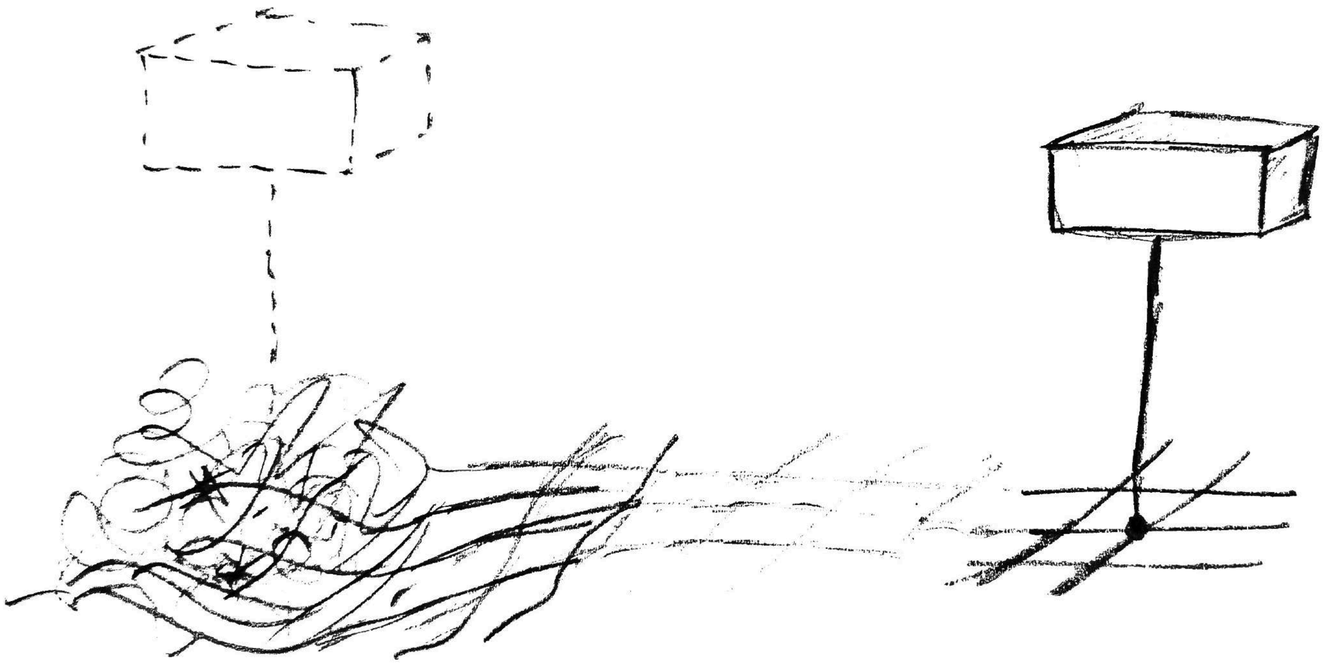


With asymptotic Flatness,
at **null infinity**,

Radiative modes
decay slower than
static modes

AND

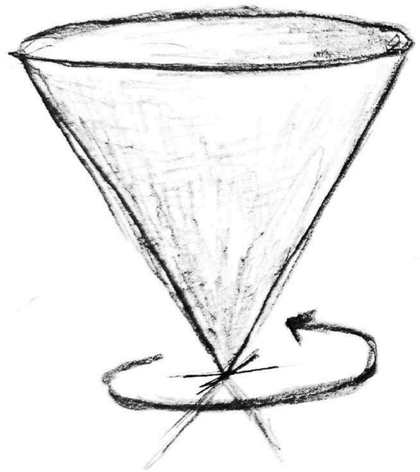
Have additional
structure for an
invariant description
of gravitational
radiation



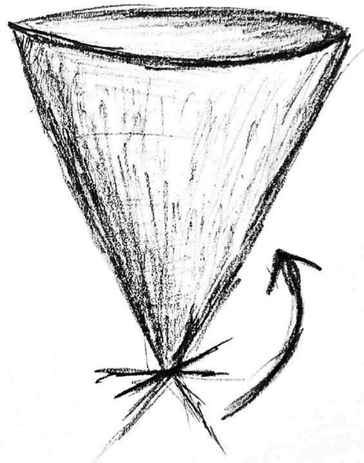
This means at
Null Infinity

GW analysis is fully non-linear and generally covariant

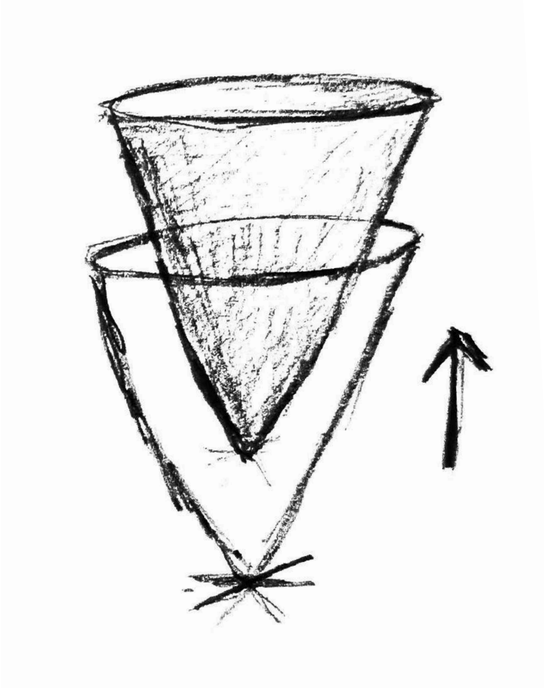
Asymptotic Symmetries: Poincare



Rotation
(real angles)



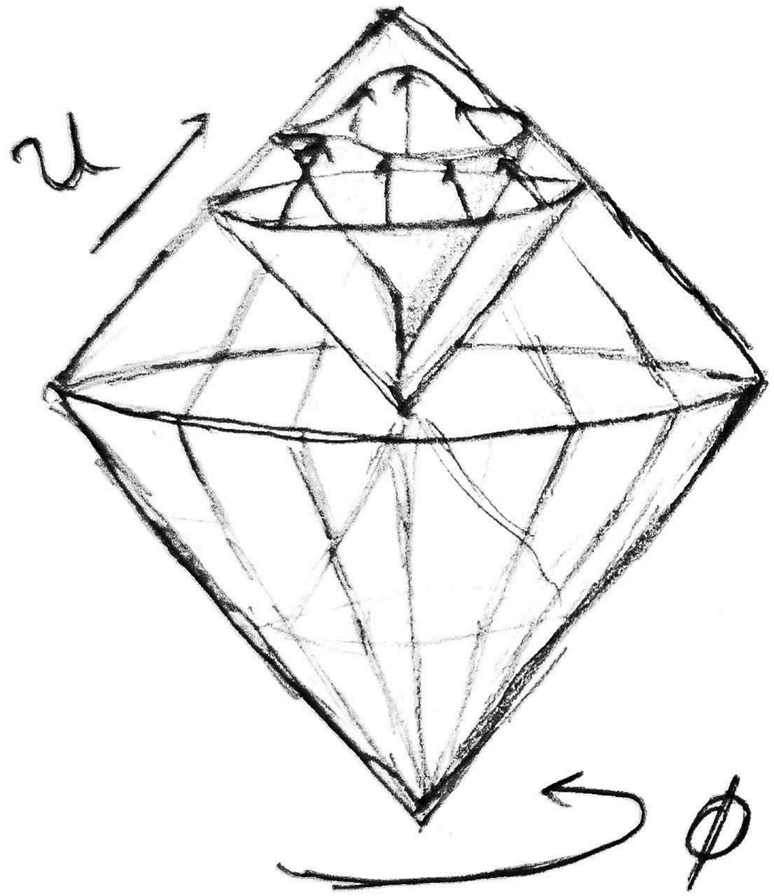
Rotation
(imaginary angles)



Rigid
Translations

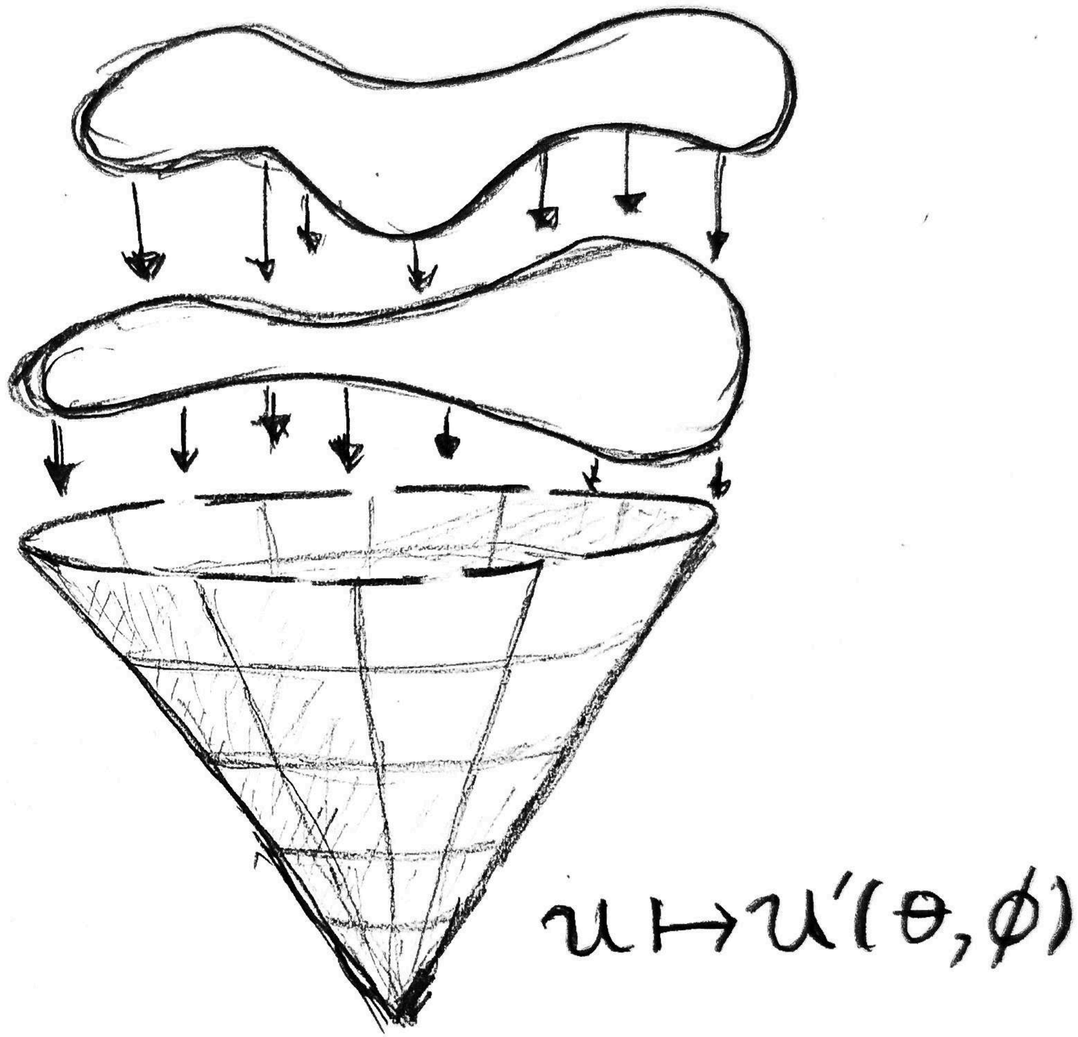
Super Translations

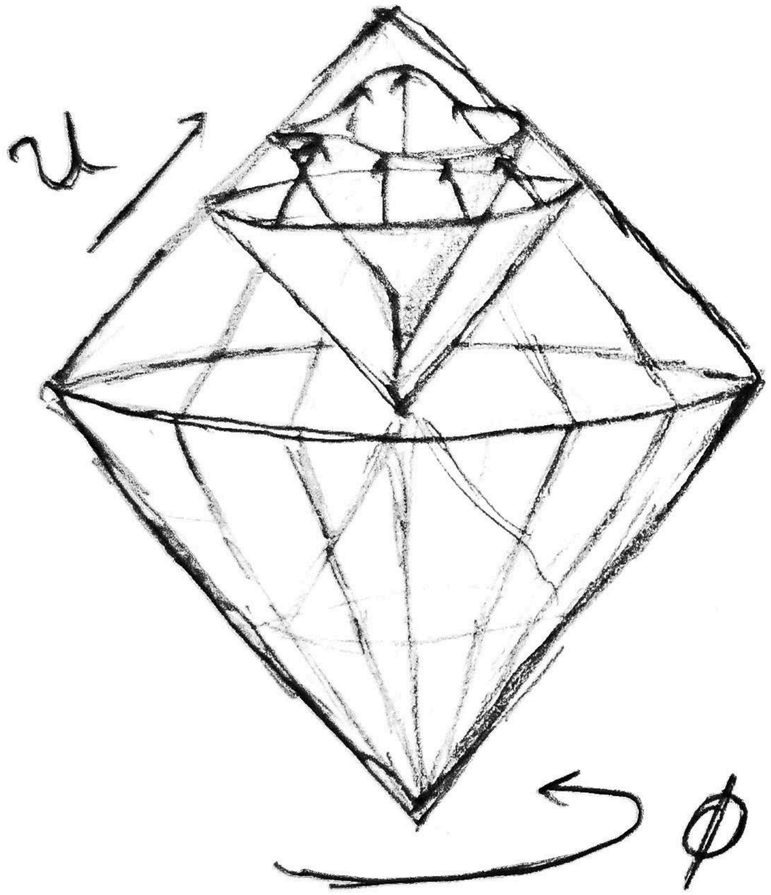
A more generalised family of translations



Super Translations

A more generalised family of translations



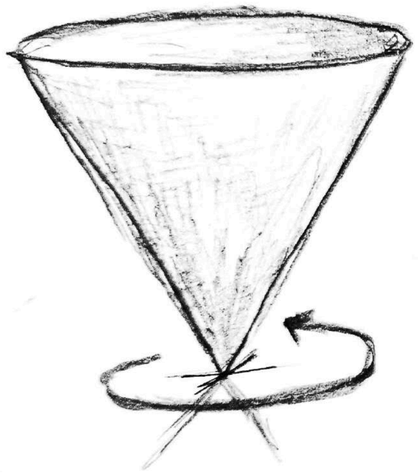


Sachs' Theorem for Super Translations

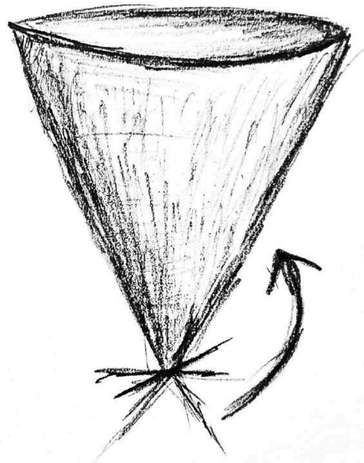
Rigid translations form a unique 4D sub-algebra.

Thus, **energy-momentum can be defined as a conjugate charge**

Asymptotic Symmetries: BMS



Rotation
(real angles)



Rotation
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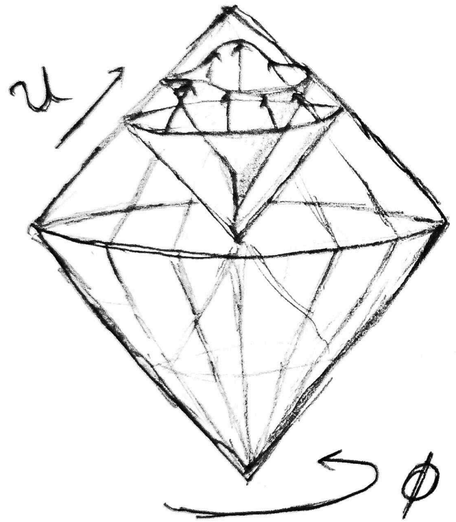
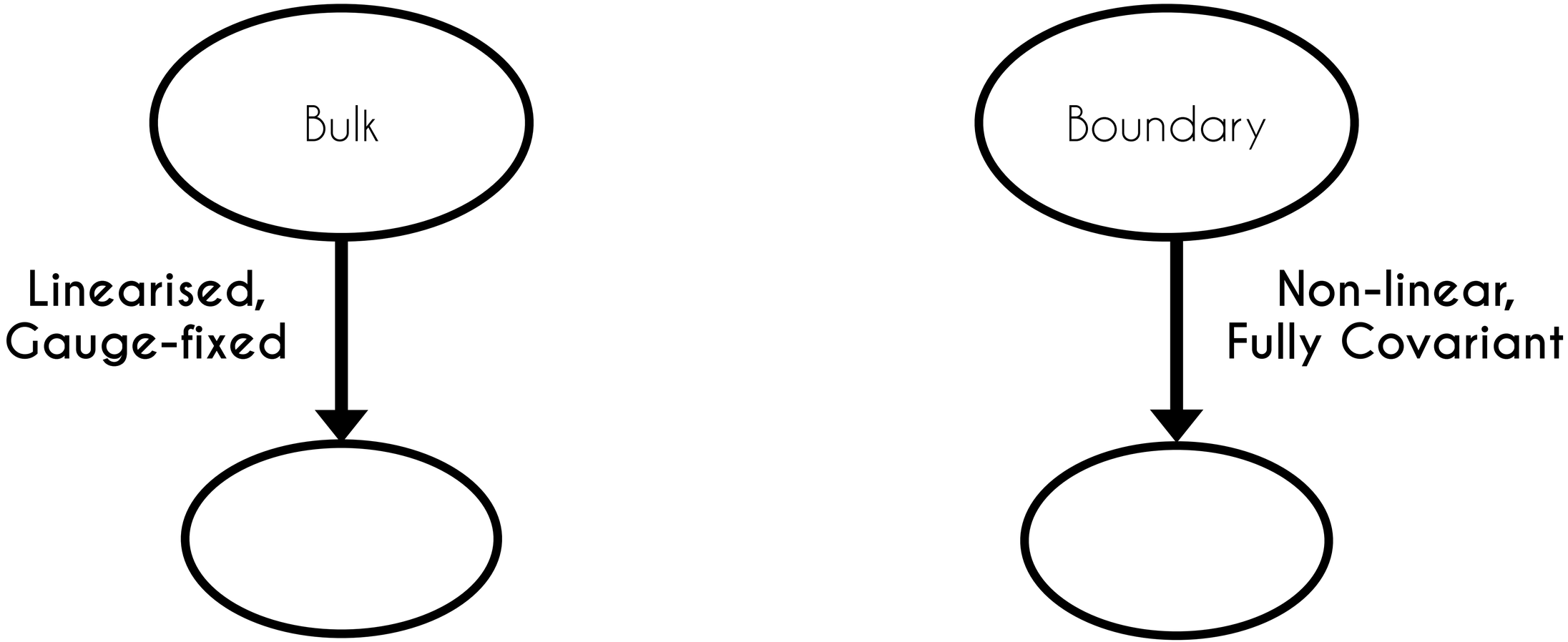


Super
Translations

Two Domains Interlocked

Two Domains Interlocked



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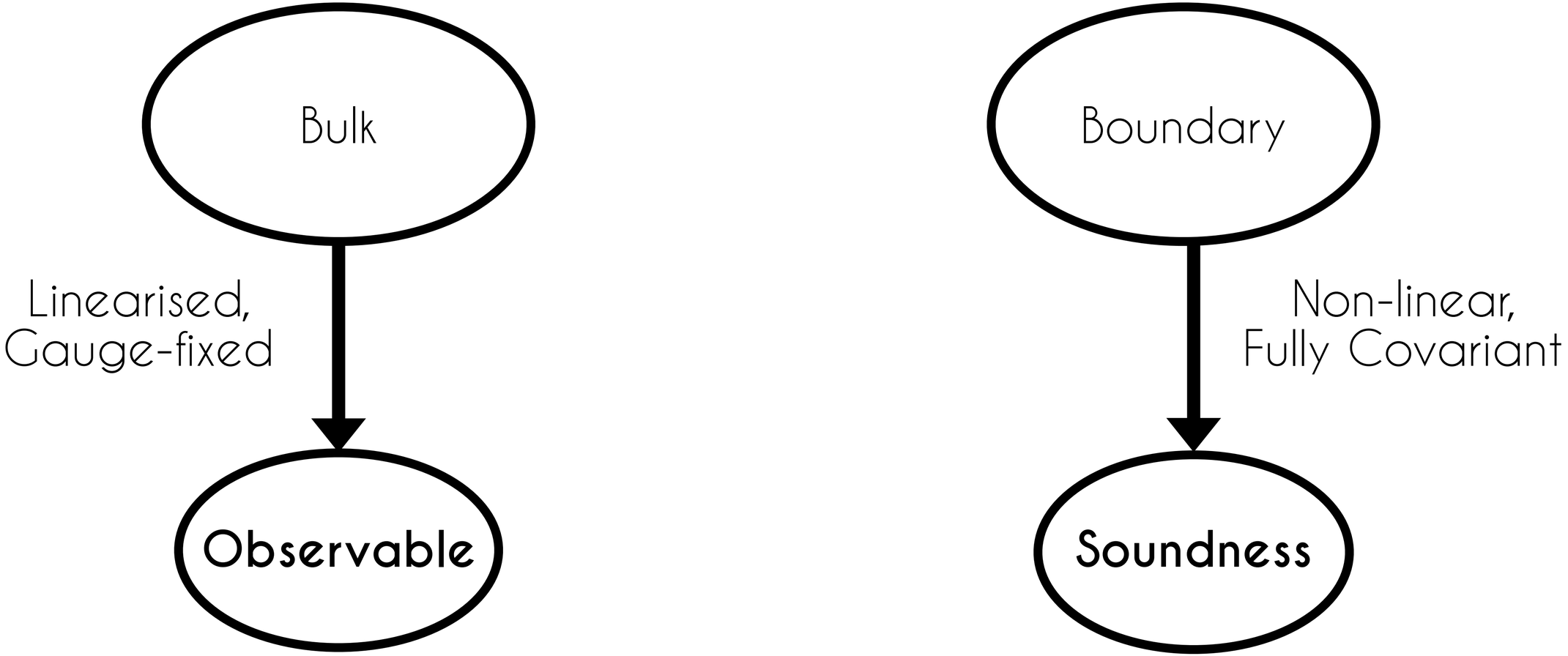

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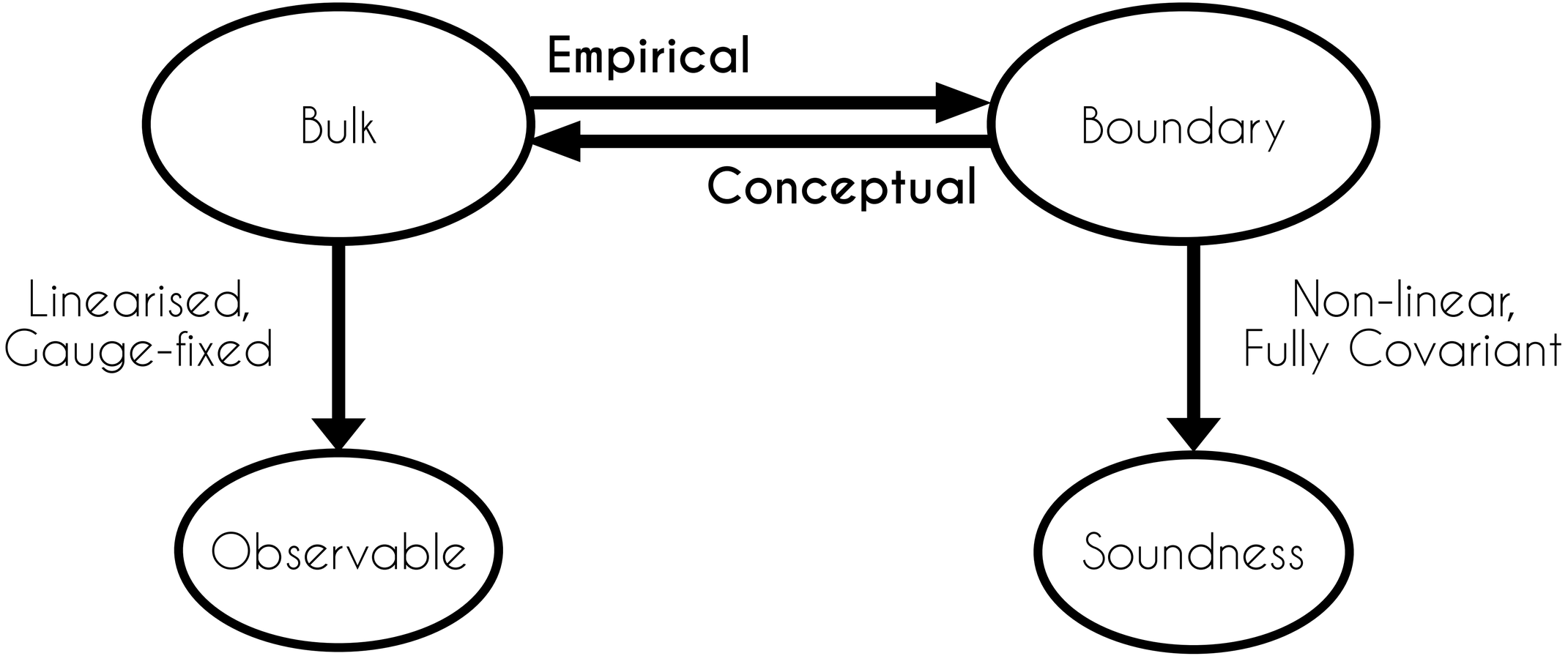

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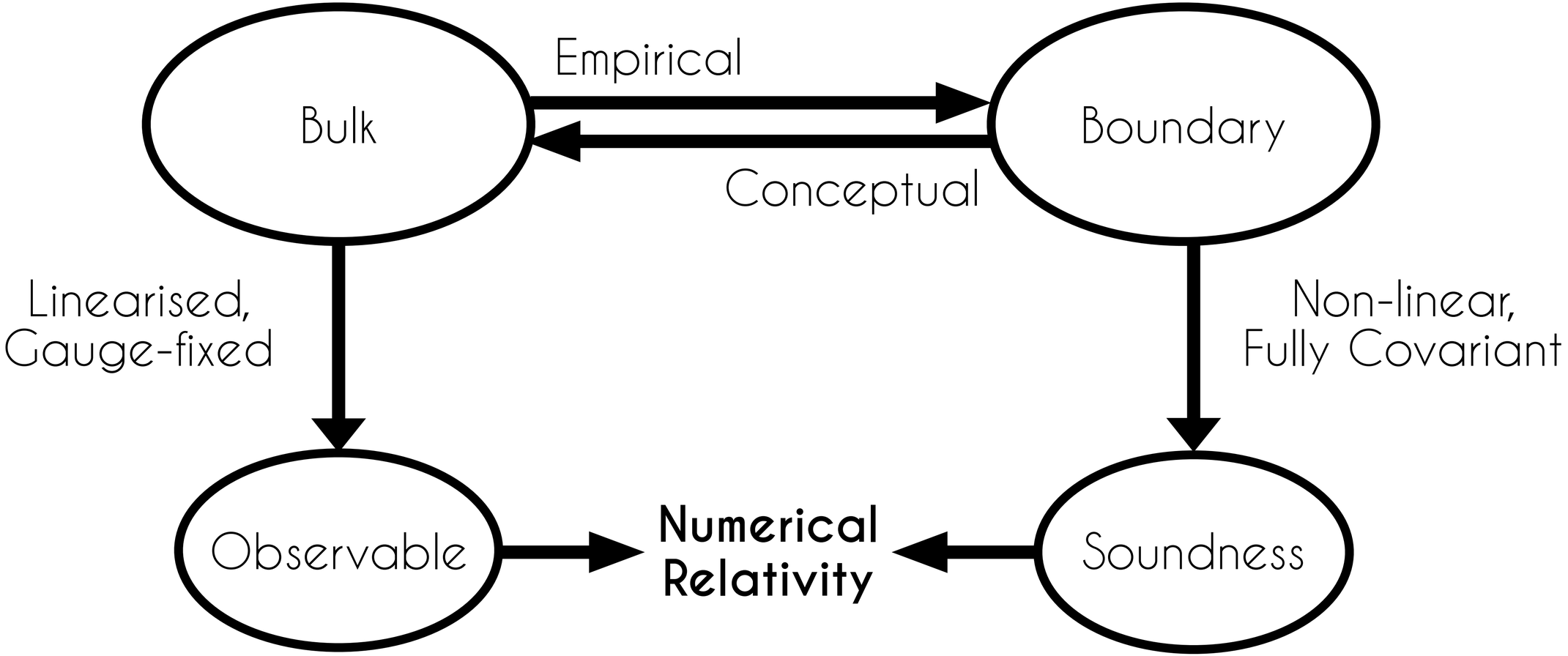
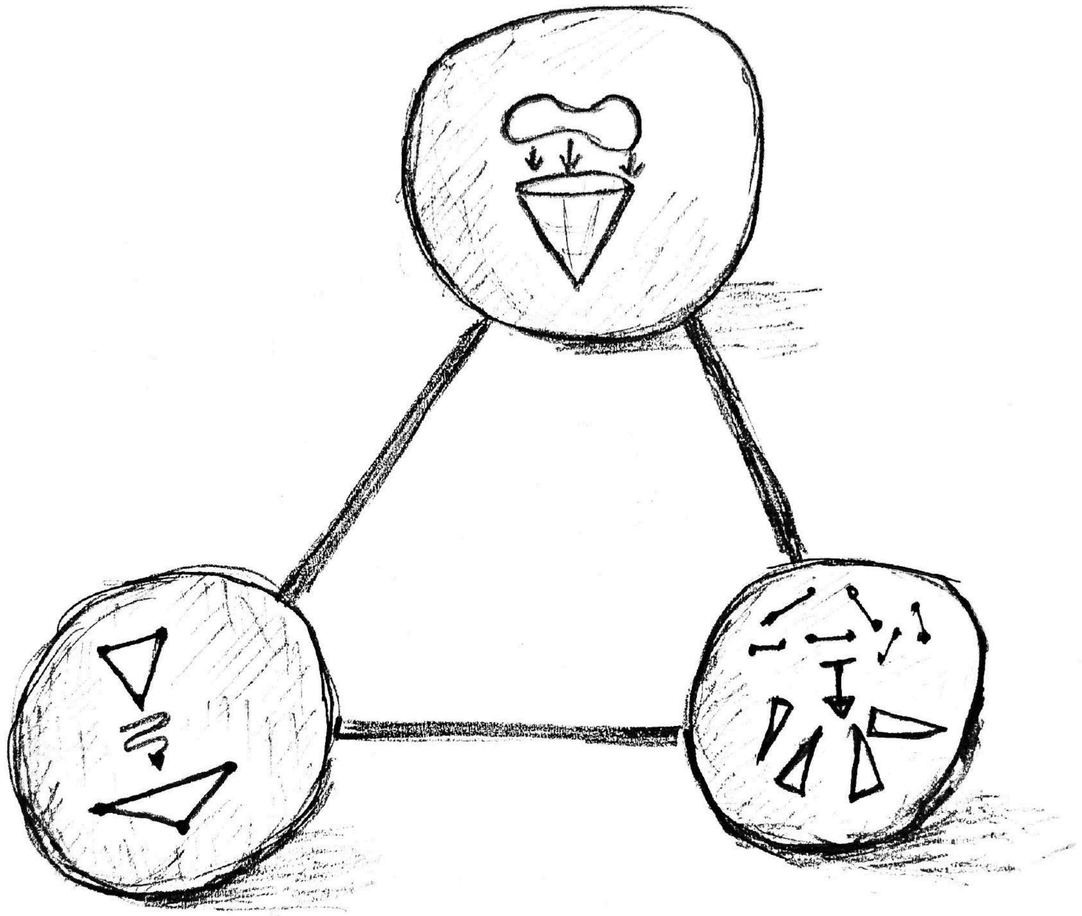


Gravitational Radiation is just one observable that receives the epistemic warrants for local calculations from concepts defined at infinity

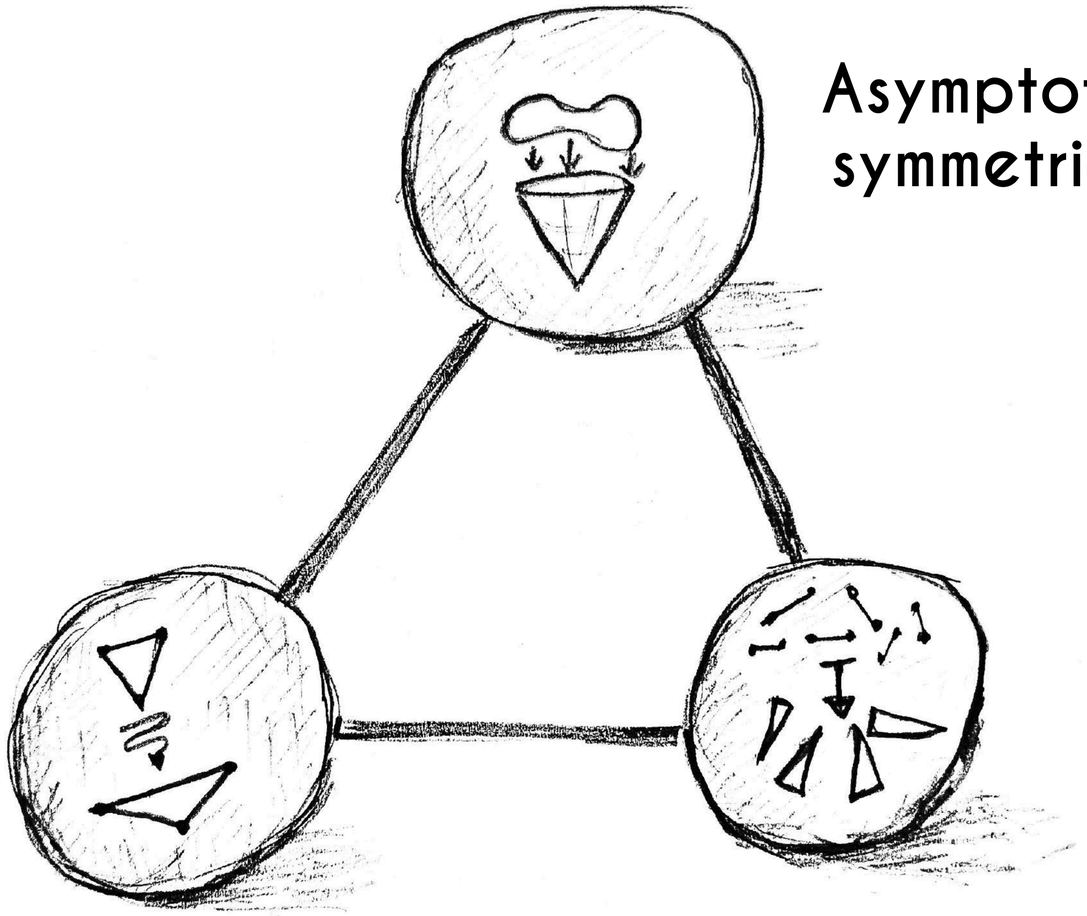
Infrared Triangle



Gravitational Radiation is just one observable that receives the epistemic warrants for local calculations from concepts defined at infinity

There are more!!!

Infrared Triangle

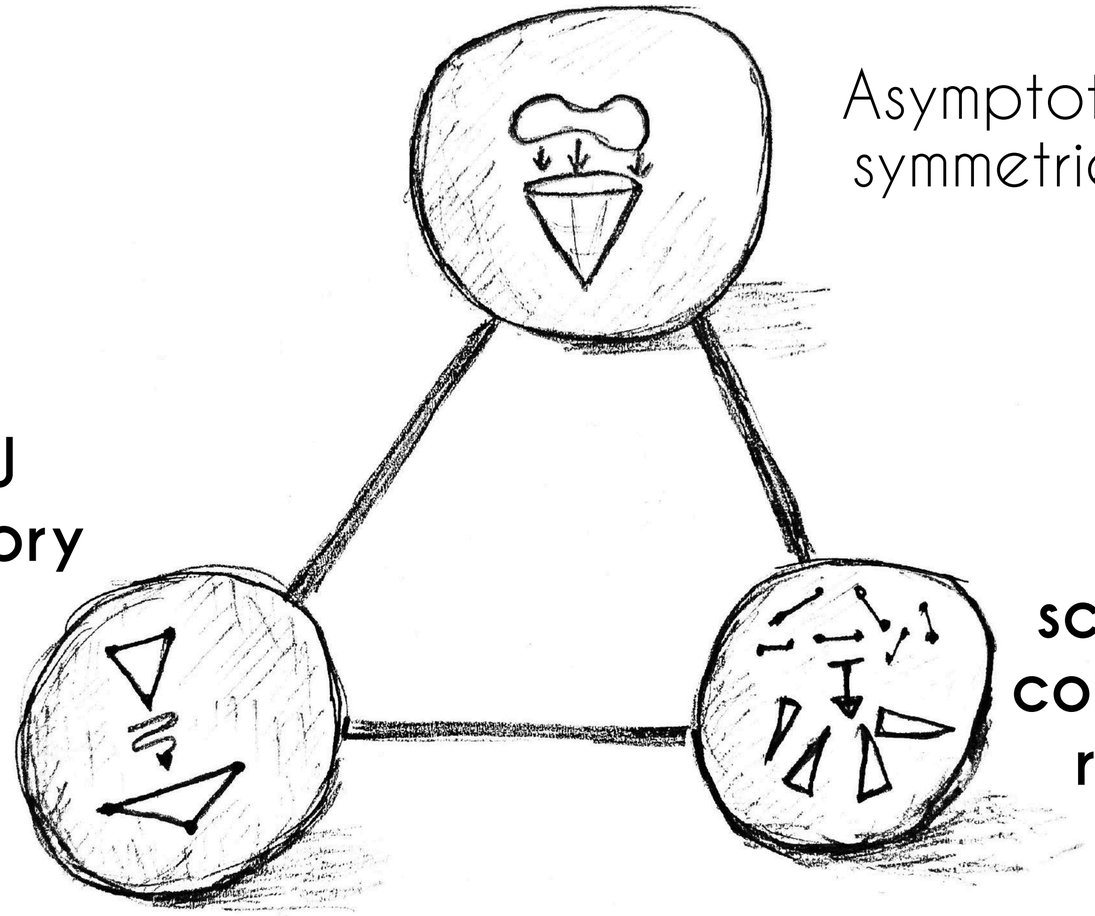


Asymptotic symmetries

Gravitational Radiation is just one observable that receives the epistemic warrants for local calculations from concepts defined at infinity

There are more!!!

Infrared Triangle



Asymptotic symmetries

GW Memory

Soft-scattering/ consistency-relations

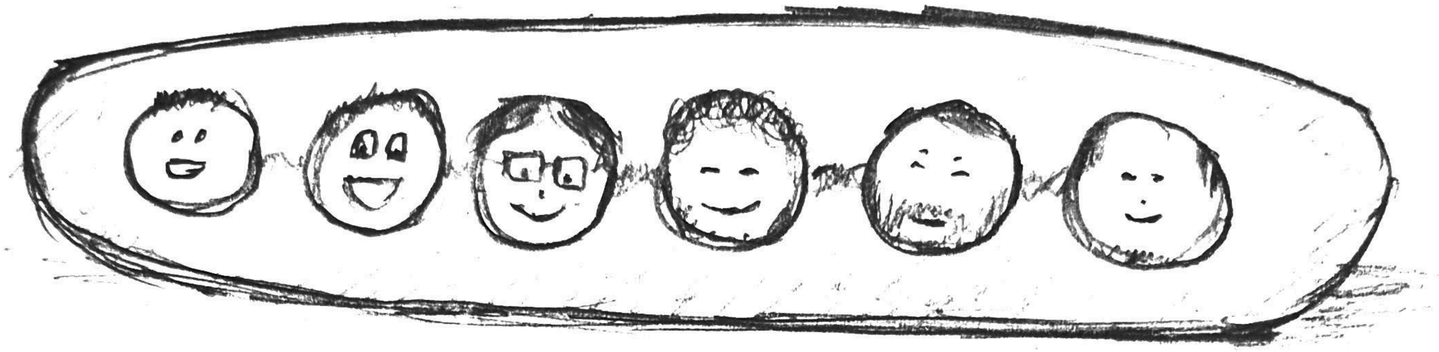
Gravitational Radiation is just one observable that receives the epistemic warrants for local calculations from concepts defined at infinity

There are more!!!

Miracle of

Assume
isolated system

Get
all this interesting physics



This framing obscures a **sensitive, contingent concord**

Concord

Conformal Completion of Asymptotic Minkowski

⇔

boundary (is *null* ∧ *infinitely far*) ∧
(*supports charge definitions* ∧ *flux balance laws*)

This framing obscures a **sensitive, contingent concord**

Concord

Conformal Completion of Asymptotic Minkowski

⇔

boundary (is ***null*** ∧ *infinitely far*) ∧
(*supports charge definitions* ∧ *flux balance laws*)

Needed for the **intrinsic geometry**

(Null normals are also transverse and provide a natural ruling)

Concord

Conformal Completion of Asymptotic Minkowski

$\Leftrightarrow$

boundary (is *null* $\wedge$ ***infinitely far***) $\wedge$
(*supports charge definitions* $\wedge$ *flux balance laws*)

Needed for the **peeling theorem**

(to isolate radiative from Coulombic components)

Concord

Conformal Completion of Asymptotic Minkowski

⇔

boundary (is *infinitely far* ∧ *null*) ∧
(*supports* ***charge definitions*** ∧ *flux balance laws*)

Needed for **identifying conserved quantities**

(for instance, a notion of energy-momentum)

Concord

Conformal Completion of Asymptotic Minkowski

⇔

boundary (is *infinitely far* ∧ *null*) ∧
(*supports charge definitions* ∧ ***flux balance laws***)

Needed for **radiation bookkeeping**

(Bondi mass-loss formula, positivity theorems, etc.)

Shattering of the Concord

The concord is **sensitive**

Shattering of the Concord

The concord is sensitive

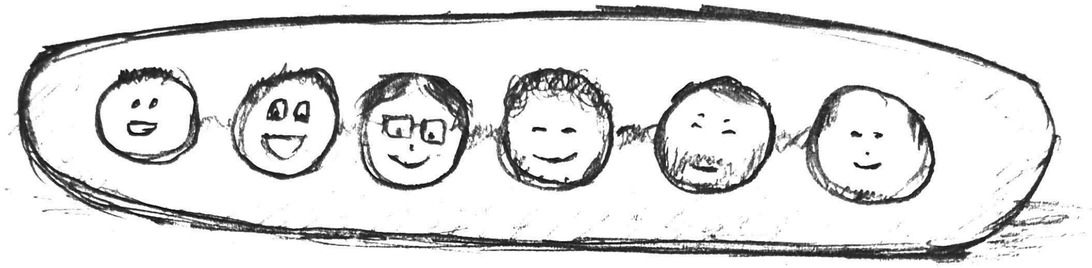
Only holds in 4 dimensions
(Hollands, Ishibashi and Wald 2016)

Shattering of the Concord

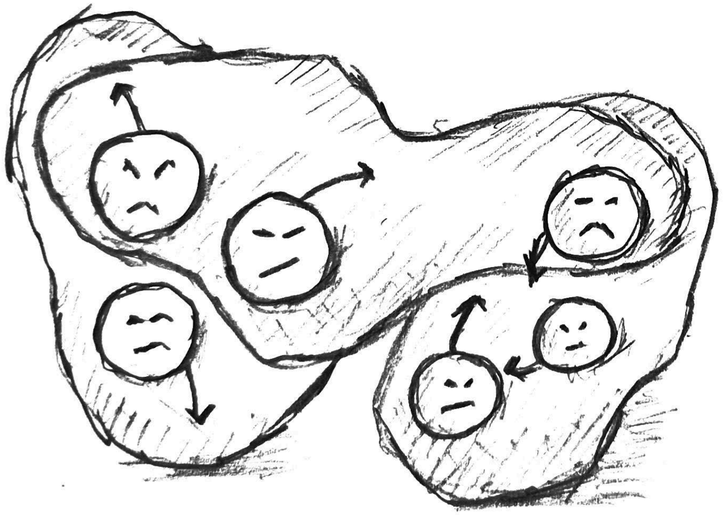
The concord is sensitive

Only holds in 4 dimensions
(Hollands, Ishibashi and Wald 2016)

Not even obtained for (physically reasonable), asymptotically-flat bulk data
(Christodoulou and Klainerman1993;
Kehrberger 2023)

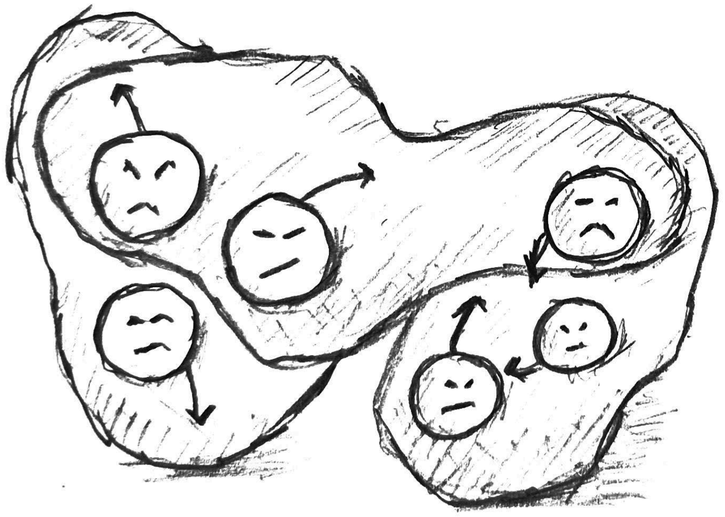


No cosmological constant



Positive cosmological constant!!!

Issues with Asymptotic de Sitter



Space-like infinity: Null rulings are lost

Weak Boundary Conditions: No interesting algebra

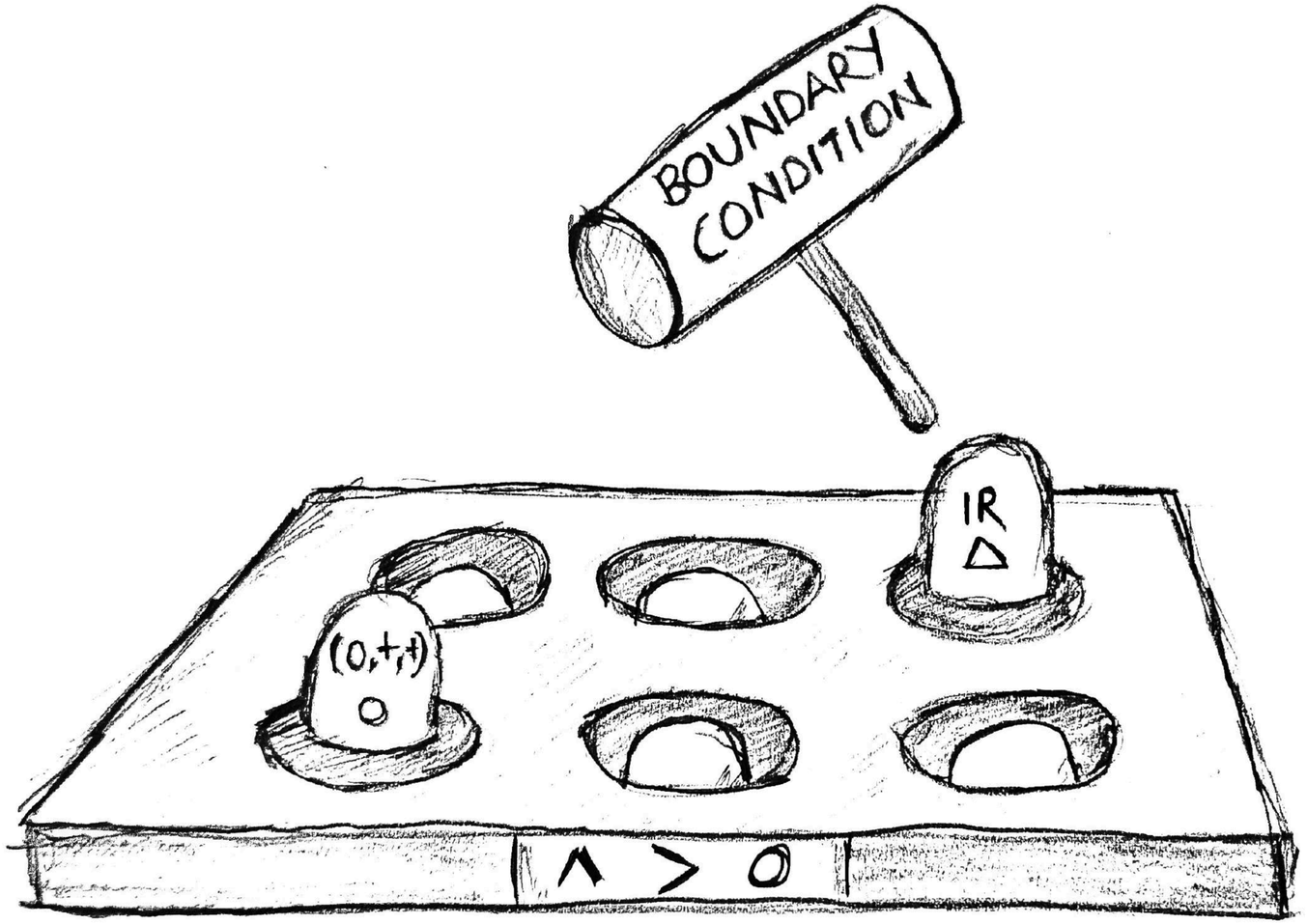
Strong Boundary Conditions: No growing modes

No Positivity Theorems (Limit discontinuous)

Backscattering: Isaacson Tensor ill-defined

The ***mere presence*** of a cosmological constant, ***no matter how small***, changes things **dramatically**

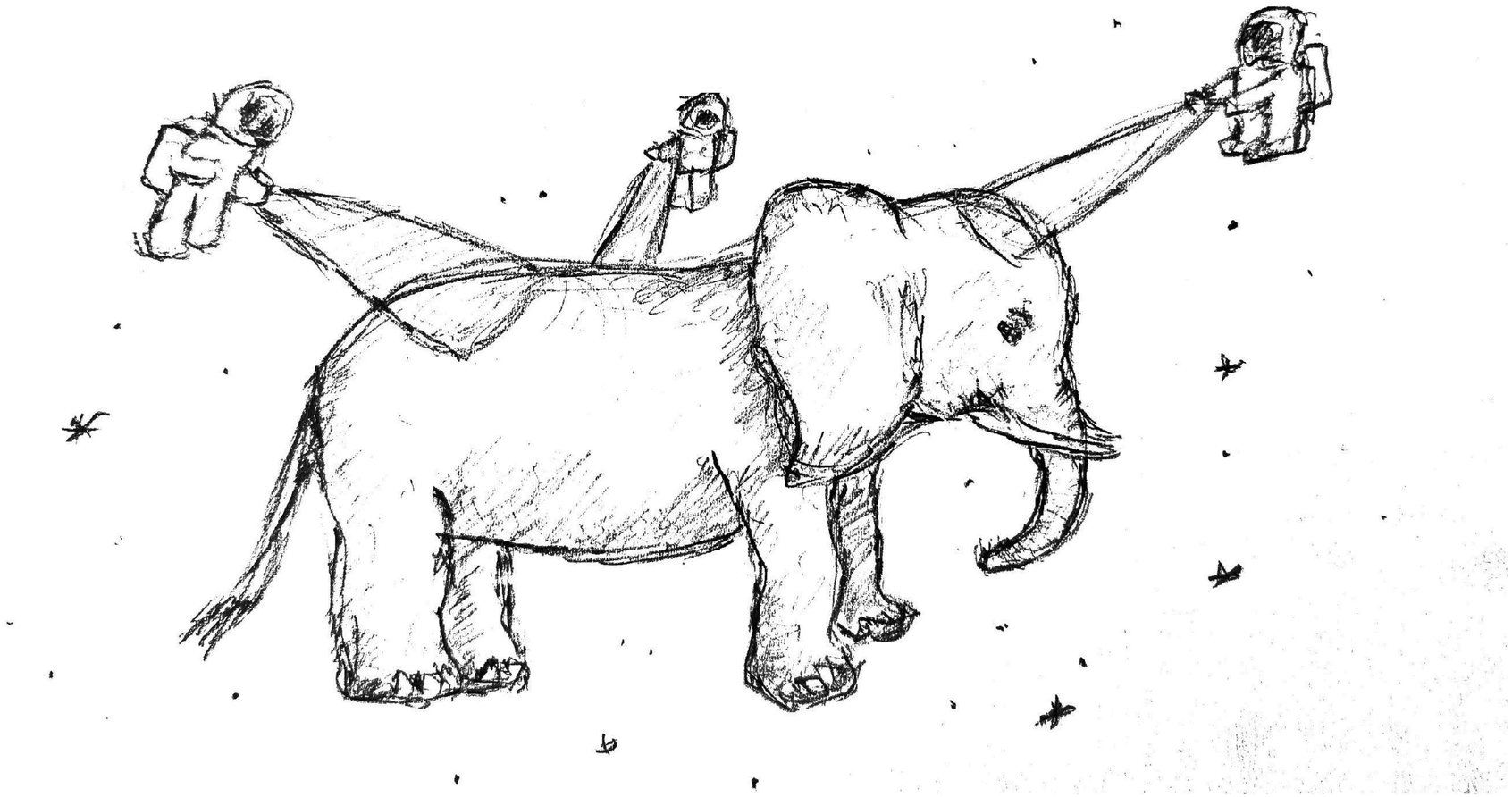
Problem of de Sitter Asymptotia

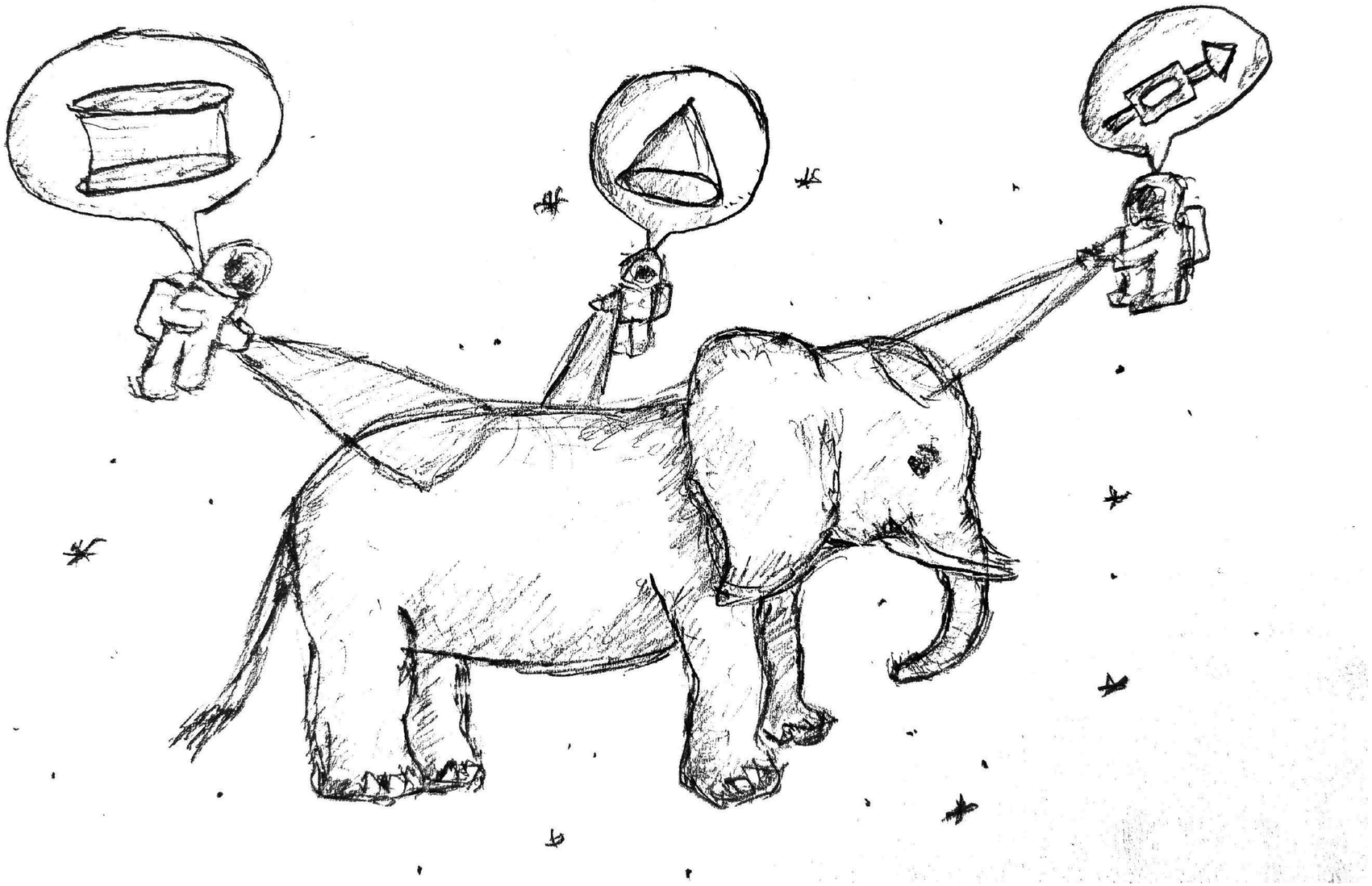


"Reasonable people may disagree on what count as reasonable boundaries"

- Stefan Hollands

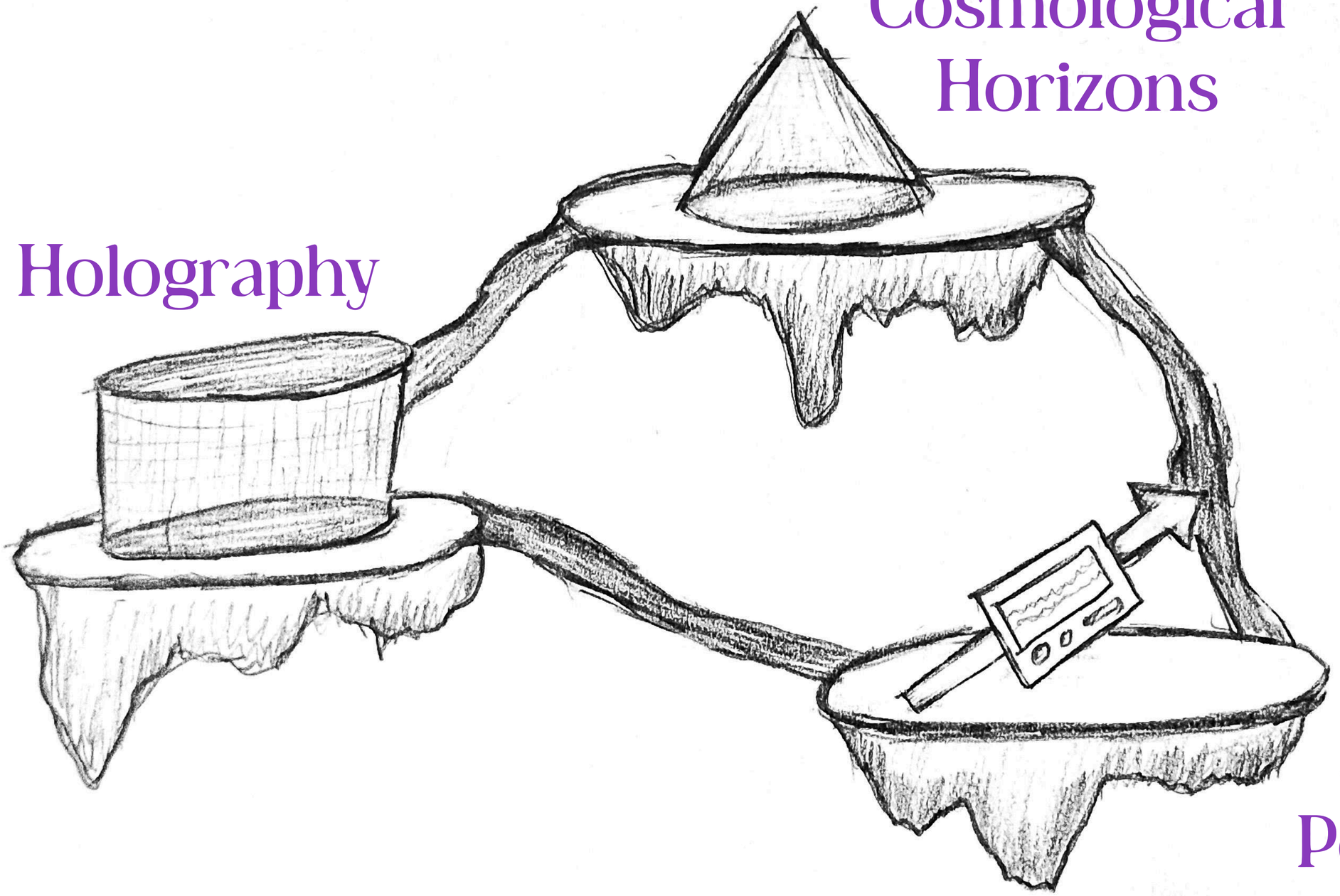
Perspectives on the Problem





Cosmological Horizons

Holography



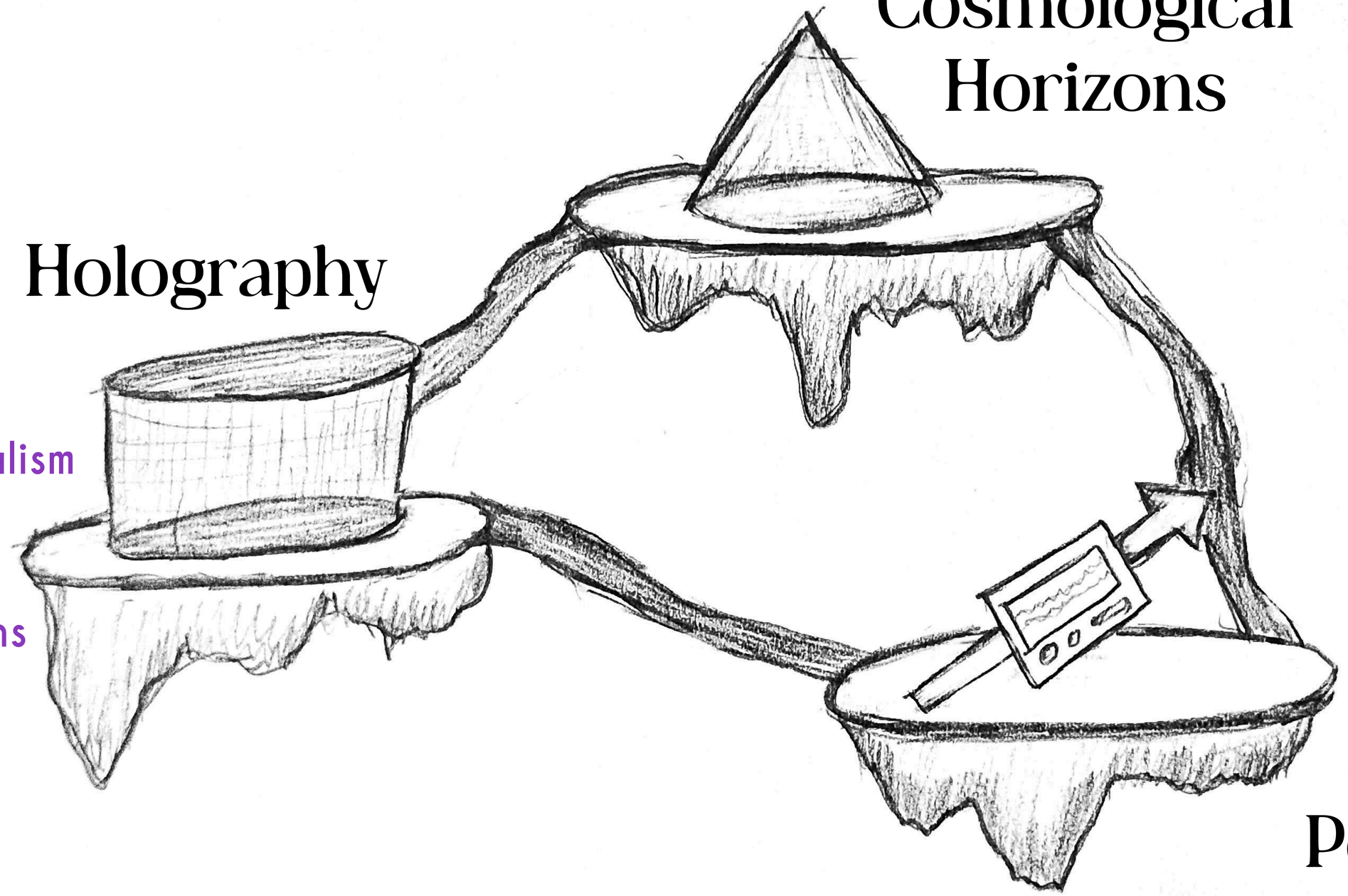
Super Poynting

Cosmological Horizons

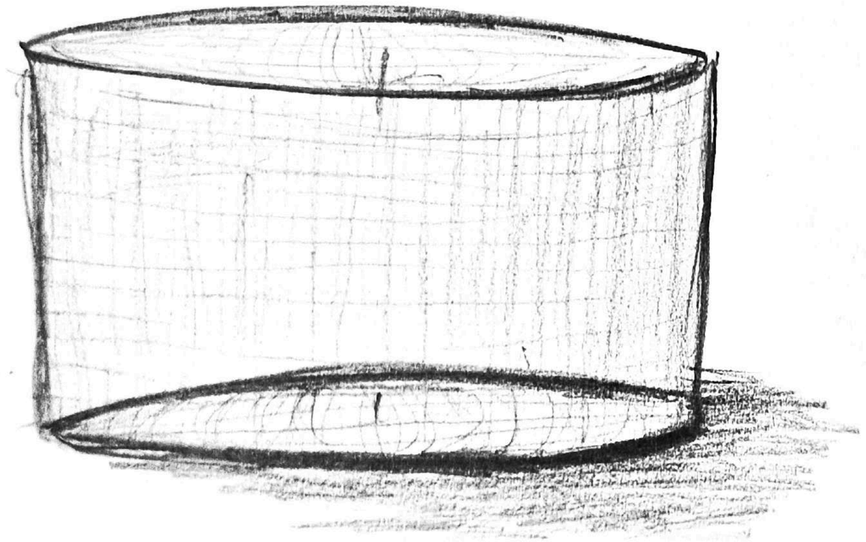
Holography

Formalism

Motivations



Super Poynting

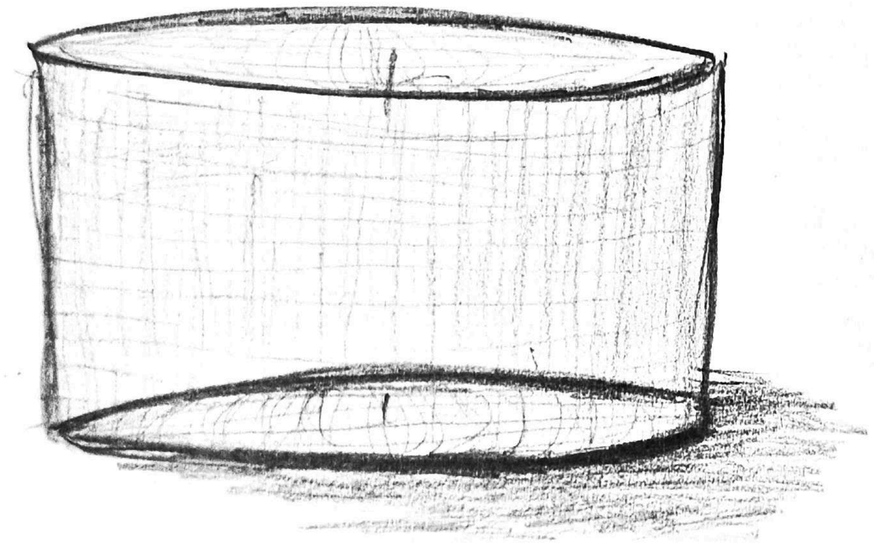


Holography (inspired)

Compere, Fiorucci, Ruzziconi (2020)
Poole, Skenderis, Taylor (2025)
McNees, Zwikel (2025)

Follow whatever smoothly limits
to the **BMS symmetry group**

1. Write metric in Bondi coordinates
2. Use Fefferman-Graham dictionary
3. Interpret analogous physics



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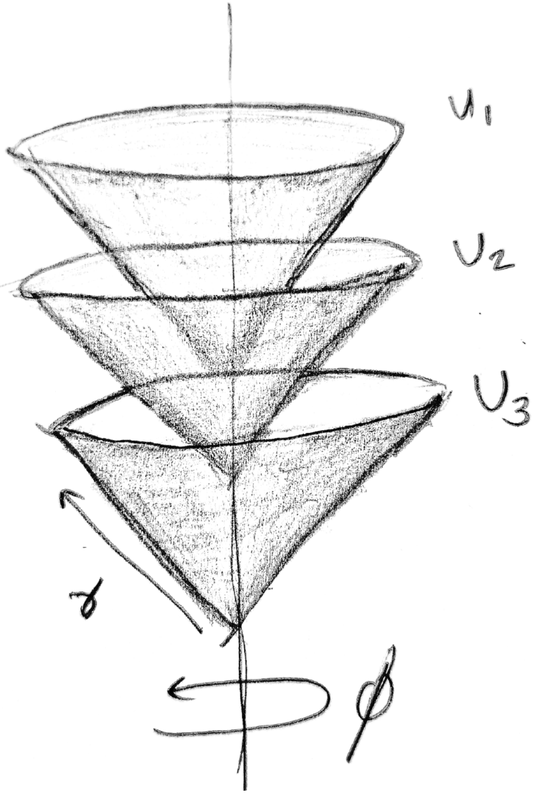
Follow whatever smoothly limits to the BMS symmetry group

Too Gauge-y(?)

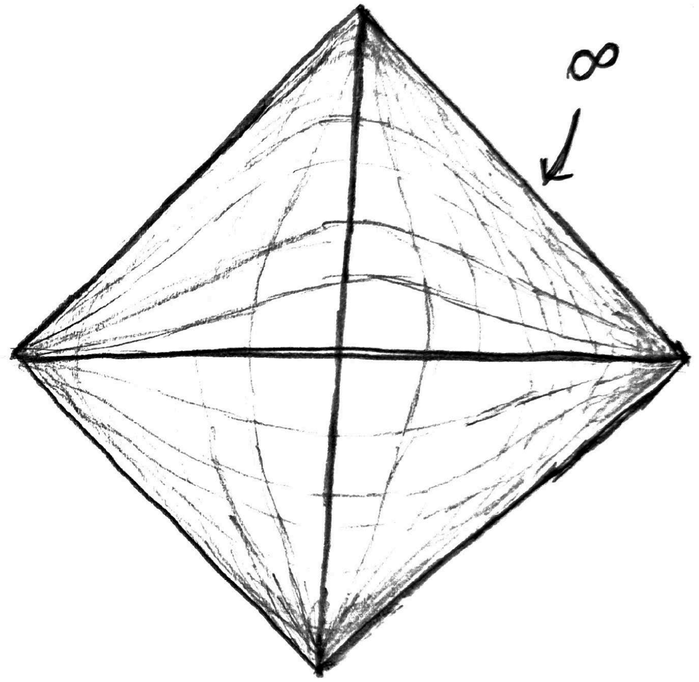
Yes, but…

Residual transformations *can* be physically meaningful



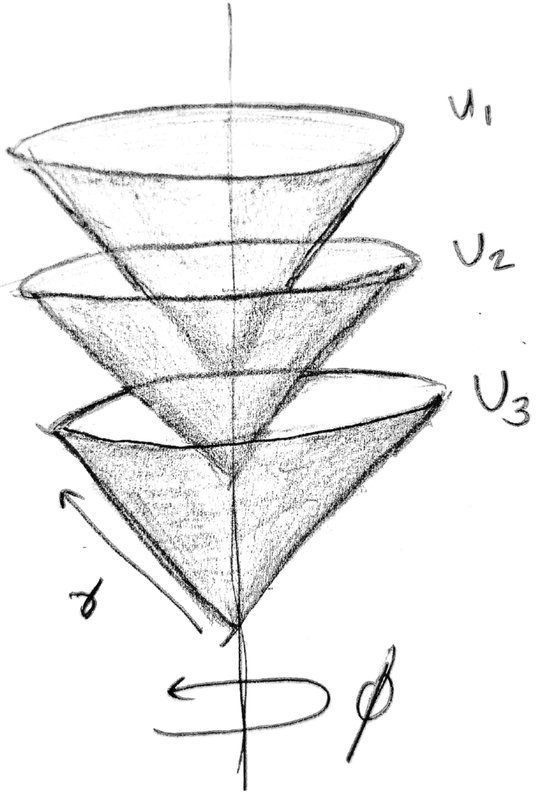
Bondi-Sachs
Coordinate
Approach

?



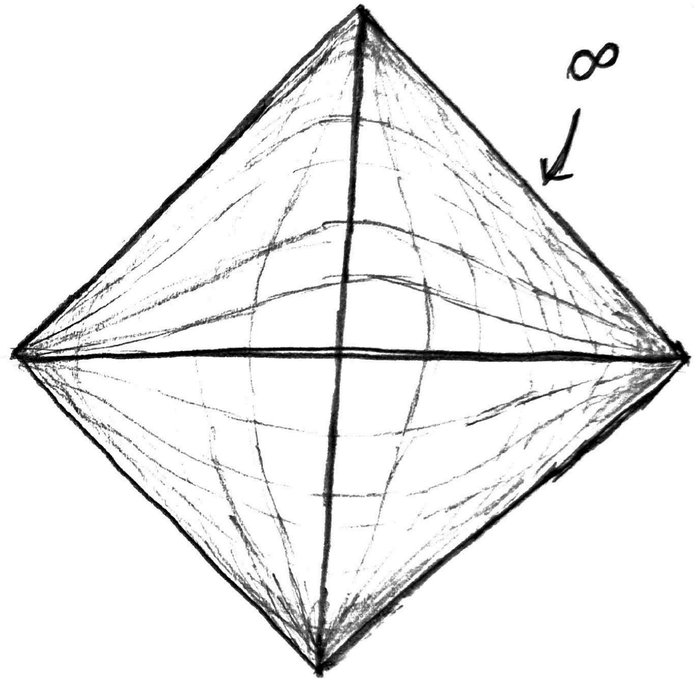
Penrose-Geroch
Geometric
Approach

Residual transformations *can* be physically meaningful

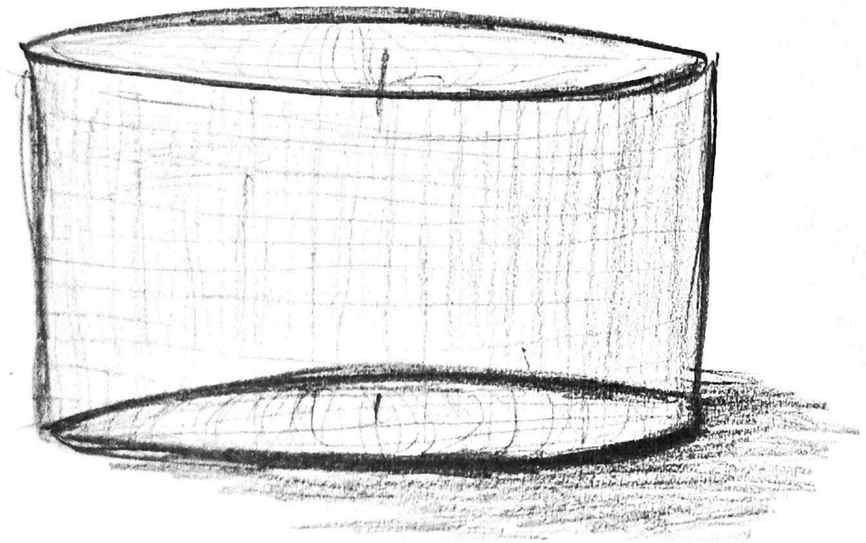


Bondi-Sachs Coordinate Approach

More epistemic legitimacy in virtue of being 'less treacherous'



Penrose-Geroch Geometric Approach



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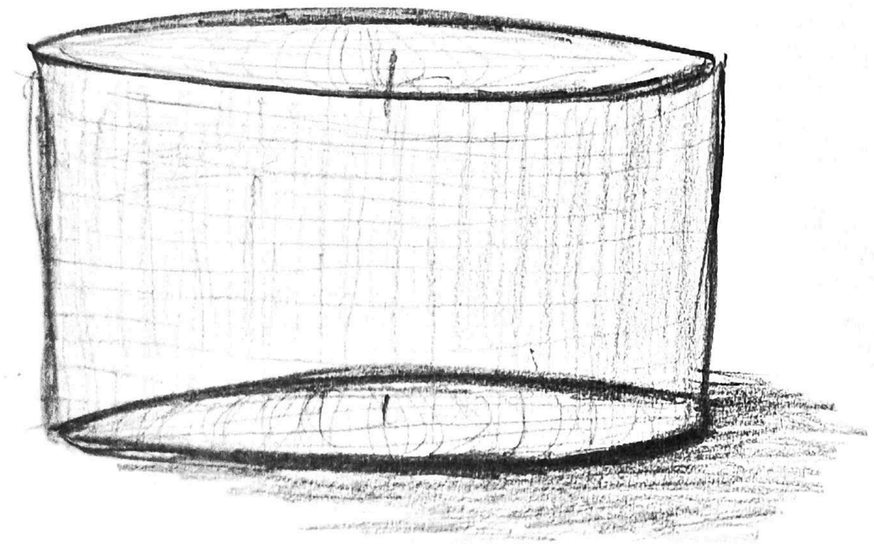
Follow whatever smoothly limits to the BMS symmetry group

Too Gauge-y(?)

Yes, but the real problem is not gauge-fixing but the lack of universal structure (i.e. algebroid instead of algebra)



Holography (inspired)

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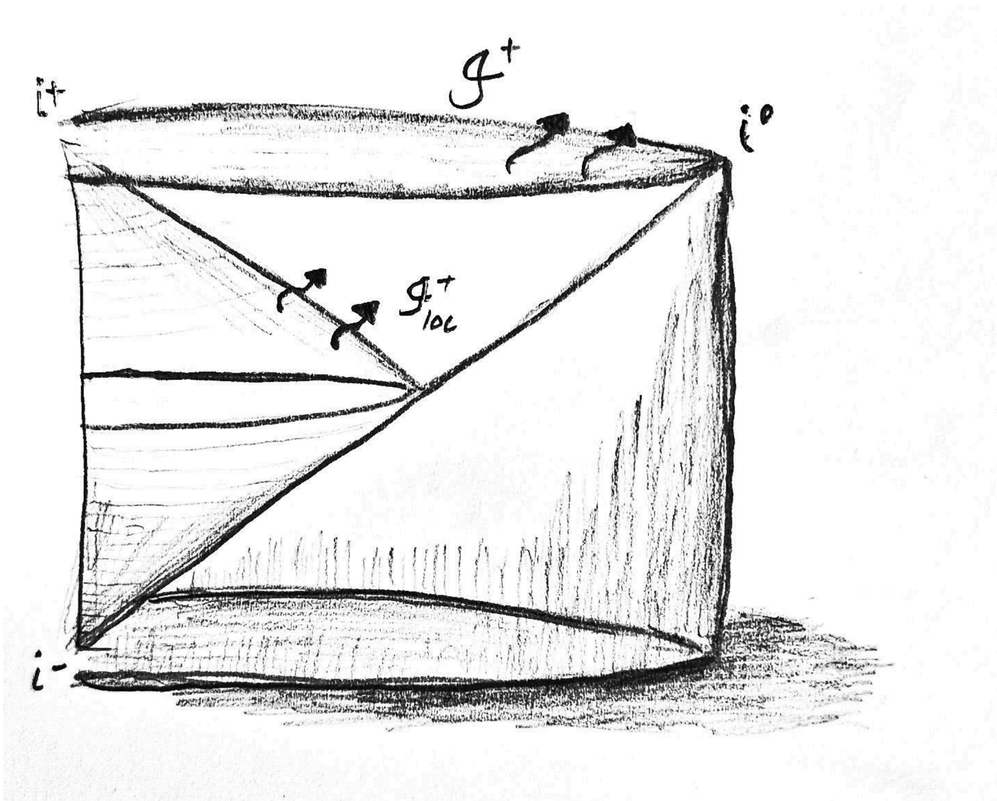
Follow whatever smoothly limits to the BMS symmetry group

Too Holographic(?)

Physical quantities interpreted through holographic renormalisation; but the schema predates holography



Cosmological Horizons

Hoque, Virmani (2018)

Ashtekar, Bahrami (2019)

Dobkowski-Rylko, Lewandowski (2020)

Use whatever privileged **null surface** one finds (cosmological horizons)

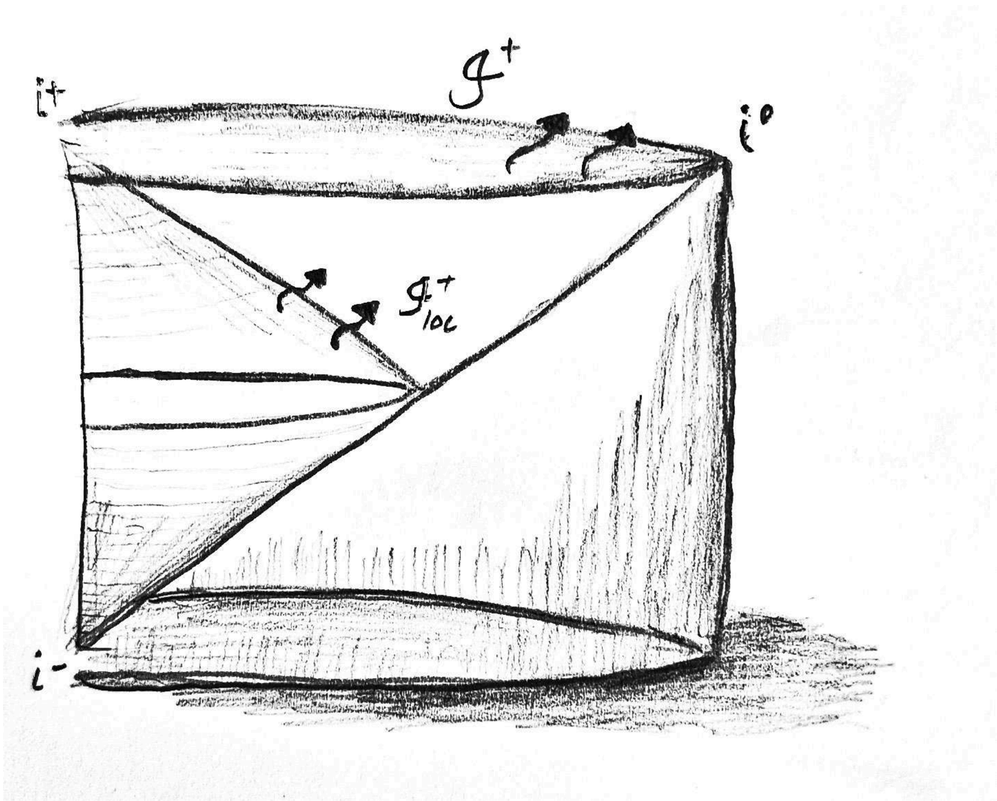
Some pleasing features

Interiorisation/No Infinity

On physical manifold

Weakly Interacting Horizon

Only causally relevant region



Cosmological Horizons

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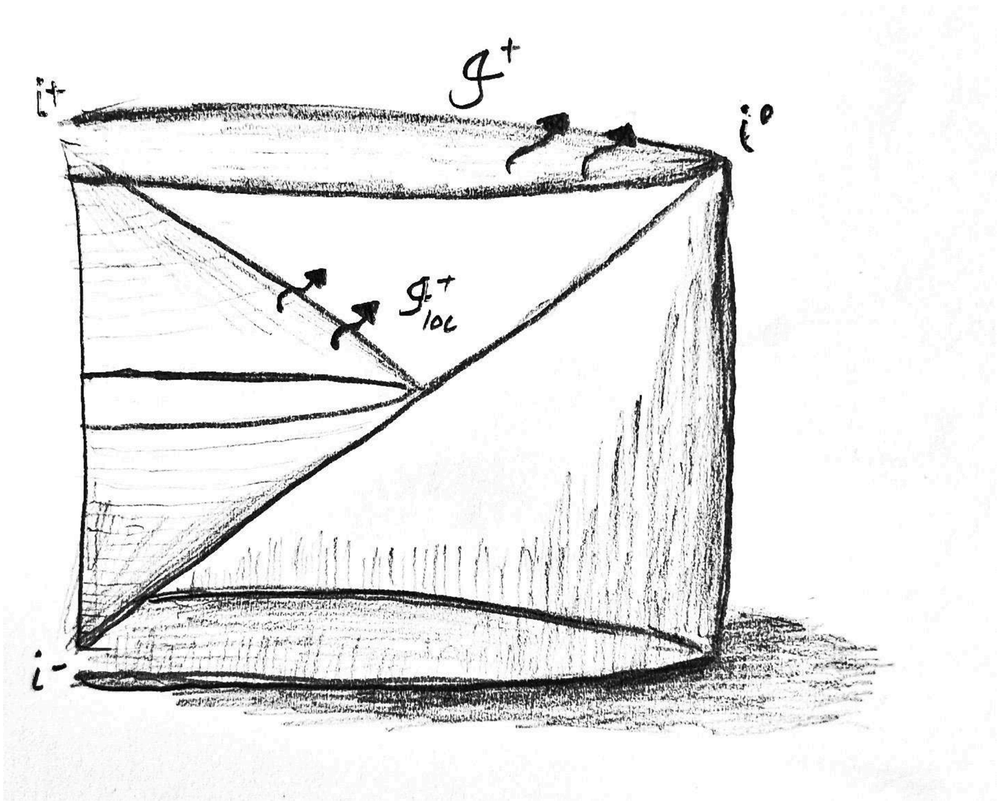
Use whatever privileged **null surface** one finds (cosmological horizons)

Furthermore

Under some assumptions, compatibility can be shown between quadrupole flux across null and space-like scri



Cosmological Horizons

Hoque, Virmani (2018)
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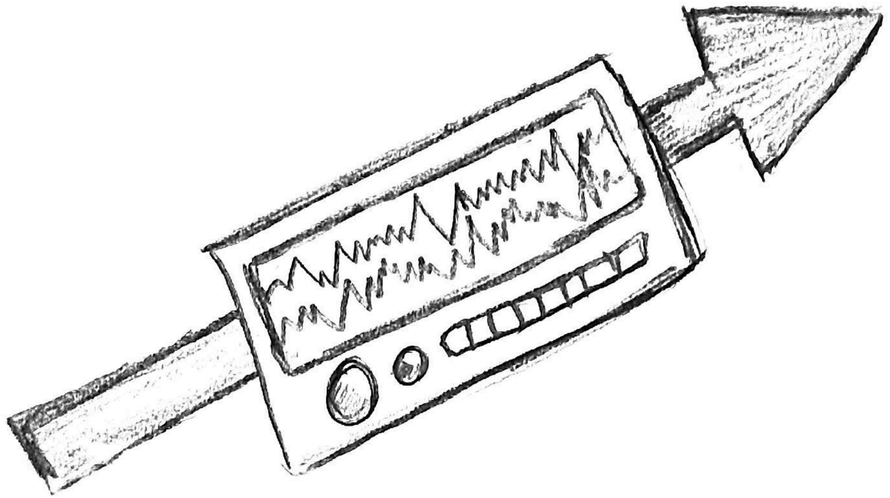
Use whatever privileged **null surface** one finds (cosmological horizons)

But ...

Analysis indexed to the observer (doesn't address global structure issues)

Completeness of local Scri+ needs to be stipulated (unclear how much it restricts the solution space)

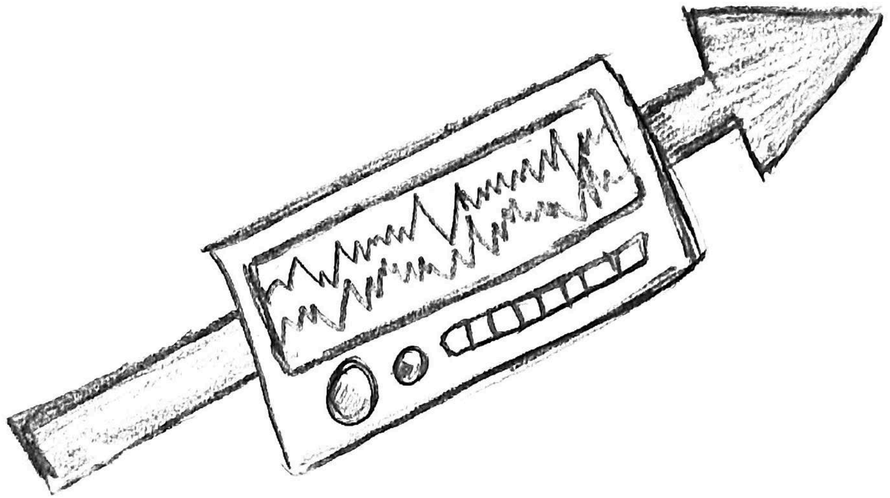


Super-Poynting

Fernandez-Alvarez, Senovilla (2020)

Follow whatever provides an
invariant radiation criterion

1. Focus on tidal strengths
2. Compute commutator of Electric and Magnetic parts of Weyl tensor



Super-Poynting

Fernandez-Alvarez, Senovilla (2020)

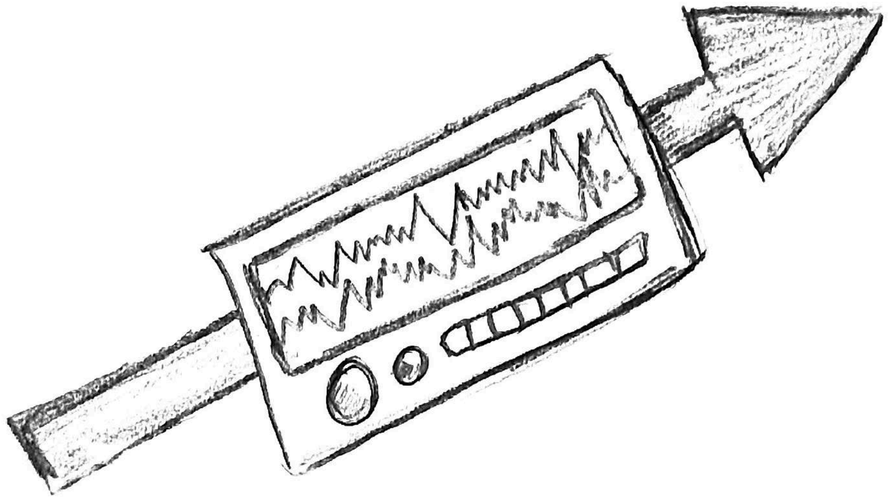
Follow whatever provides an invariant radiation criterion

Fruitfulness?

Positivity, Stability Theorems, DEC… vs
Charges identification, IR Observables…

Fundamentality?

Primacy of super-energetic accounts over energetic ones in classical GR



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Super-Poynting

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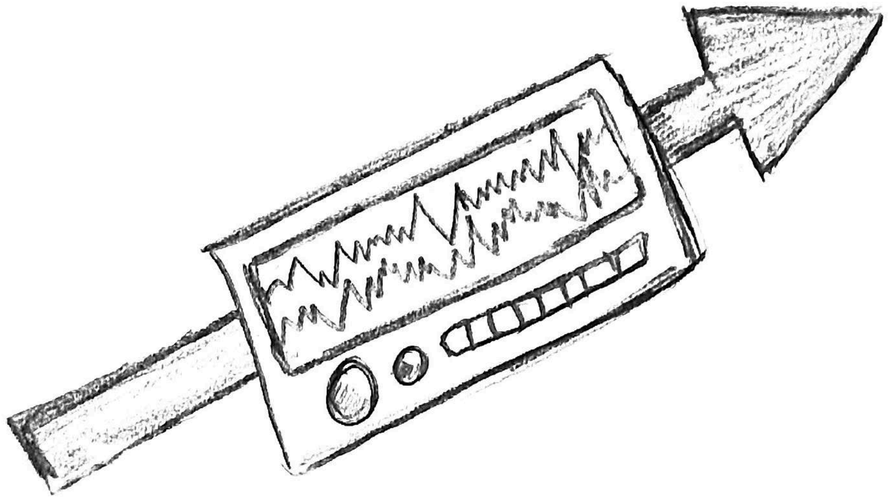
Compatibility of Criterion

- Written in terms of holographic variables
- Computed across Weakly Isolated Horizons

Maybe, but...



Super-Poynting

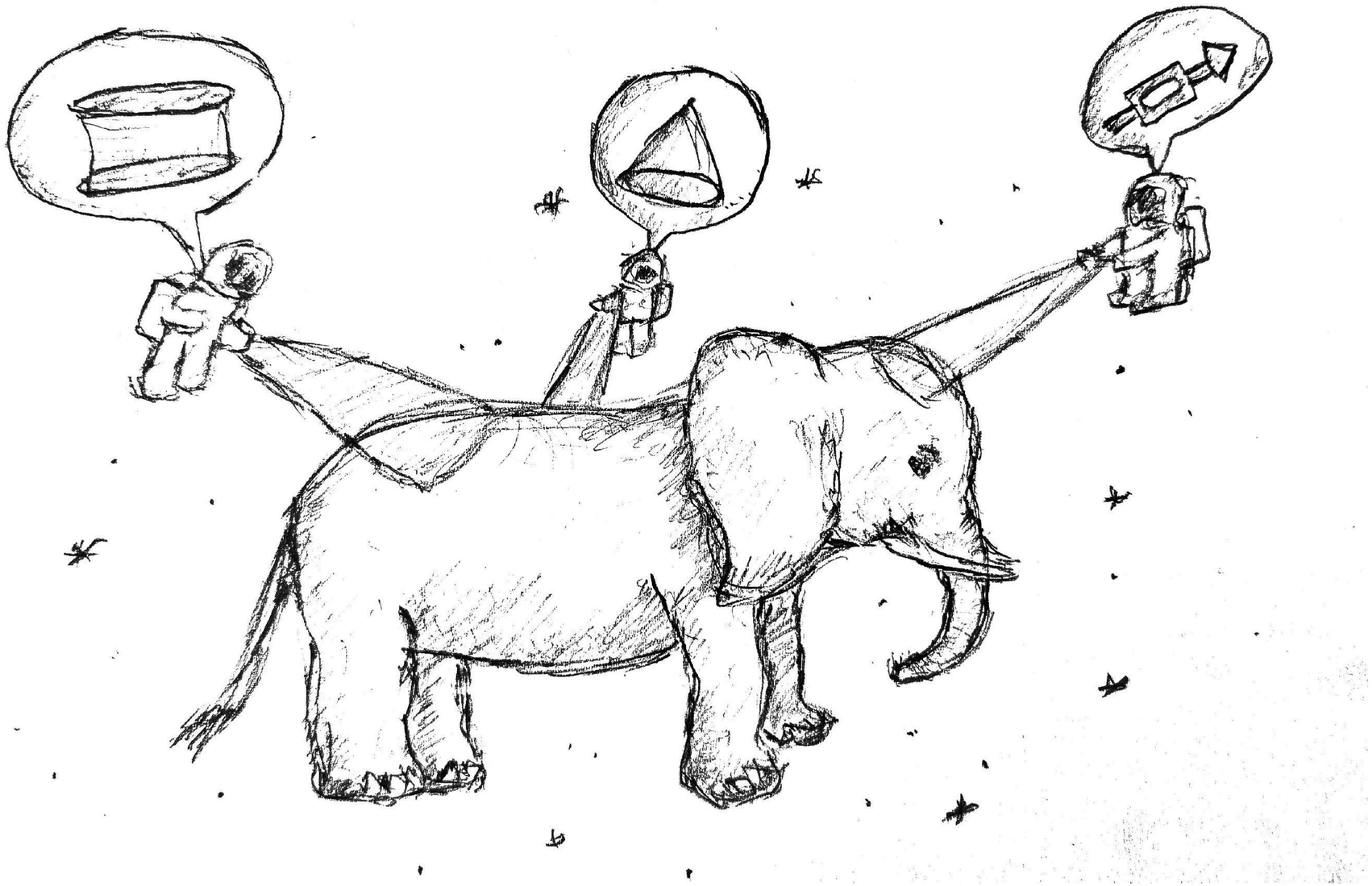
Fernandez-Alvarez, Senovilla (2020)

Follow whatever provides an
invariant radiation criterion

Compatibility of Criterion

- Written in terms of holographic variables
- Computed across Weakly Isolated Horizons

Maybe, but the criterion isn't strong enough



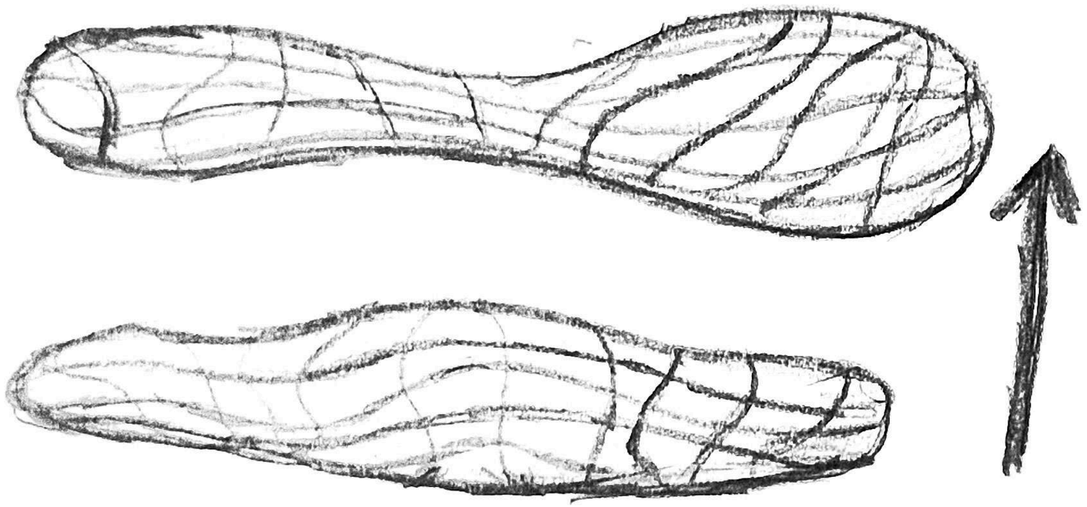
Abstract problem. Bah!

No bearing on observable phenomena.

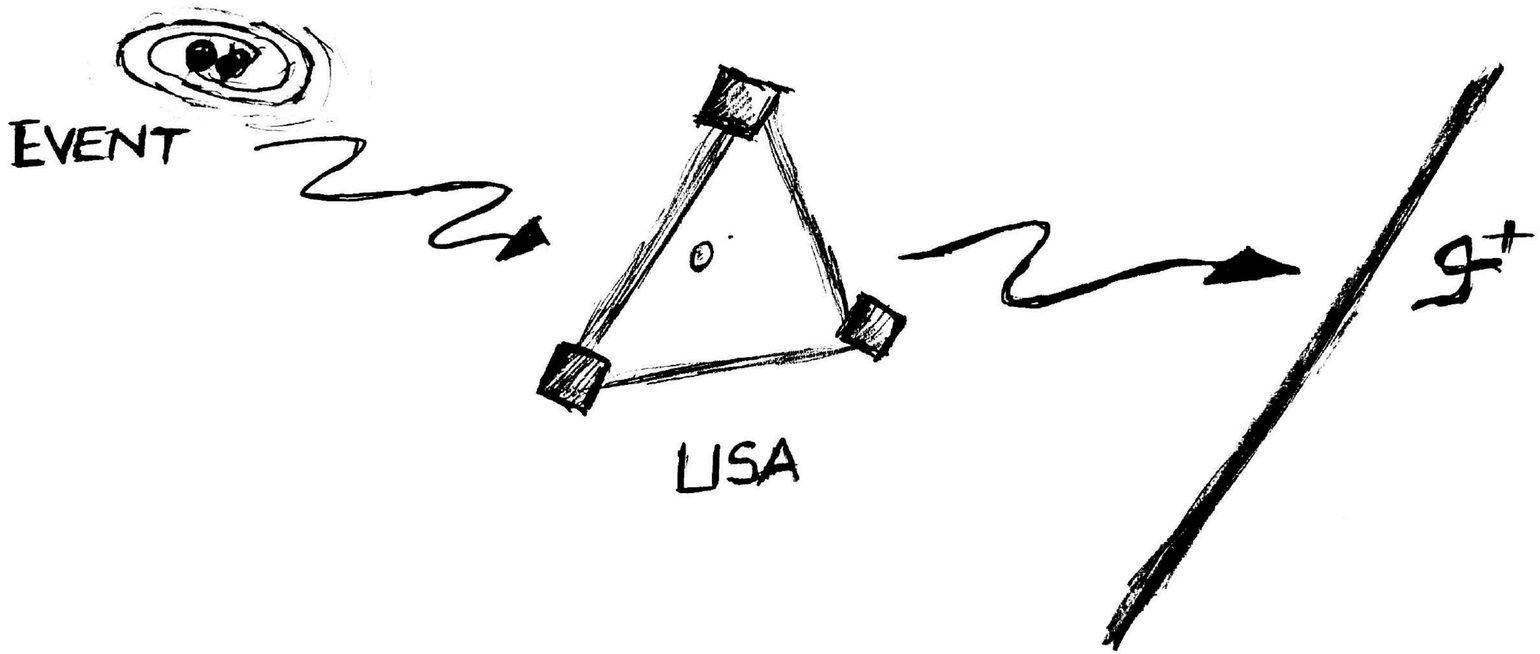
Gravitational Wave Memory

Super-translation between degenerate vacua states within the equivalence class

Permanent distortion in matter structures !!!



Is **Asymptotic Flatness** a Controlled Idealisation or Constitutive Element



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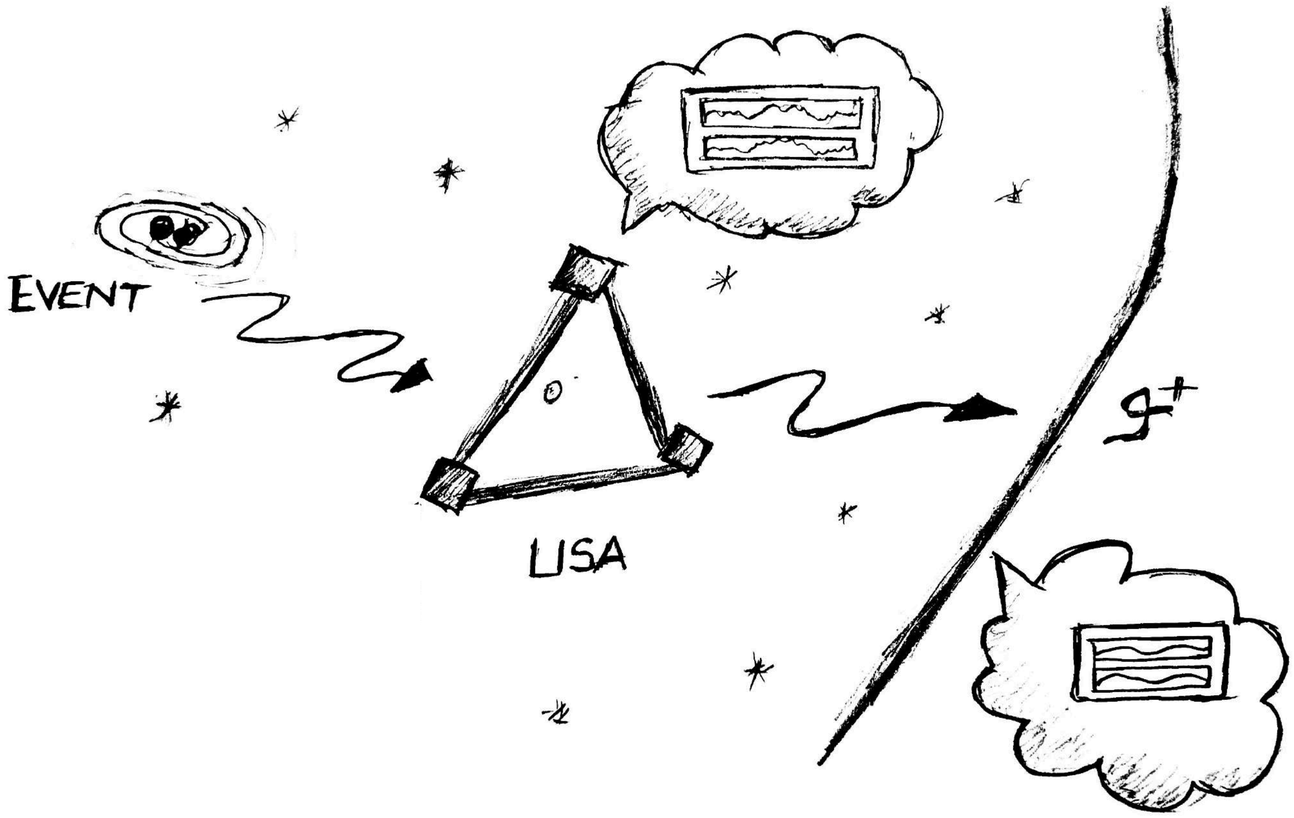

Controlled idealisation

(i.e. an idealisation with epistemic control/de-idealisation procedure)

Is **Asymptotic Flatness** a
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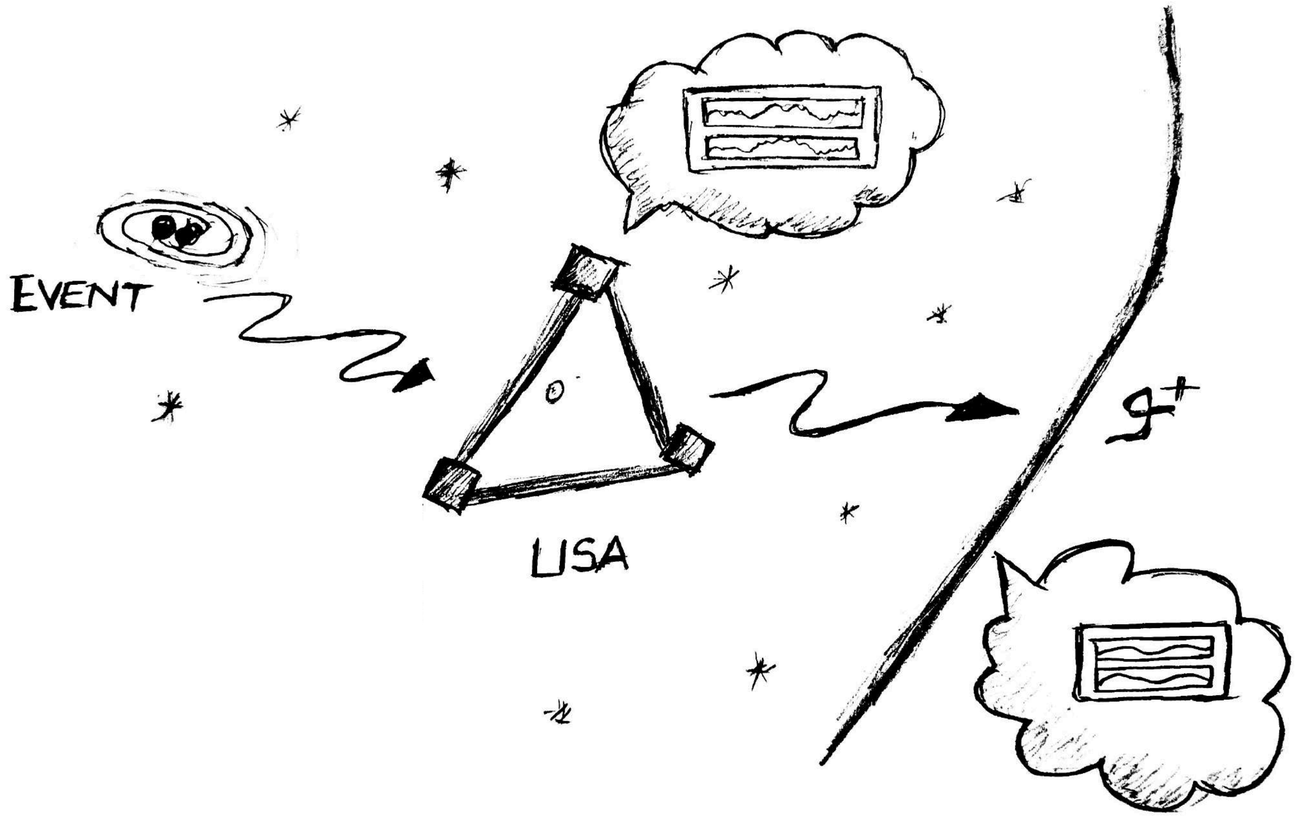


Controlled idealisation

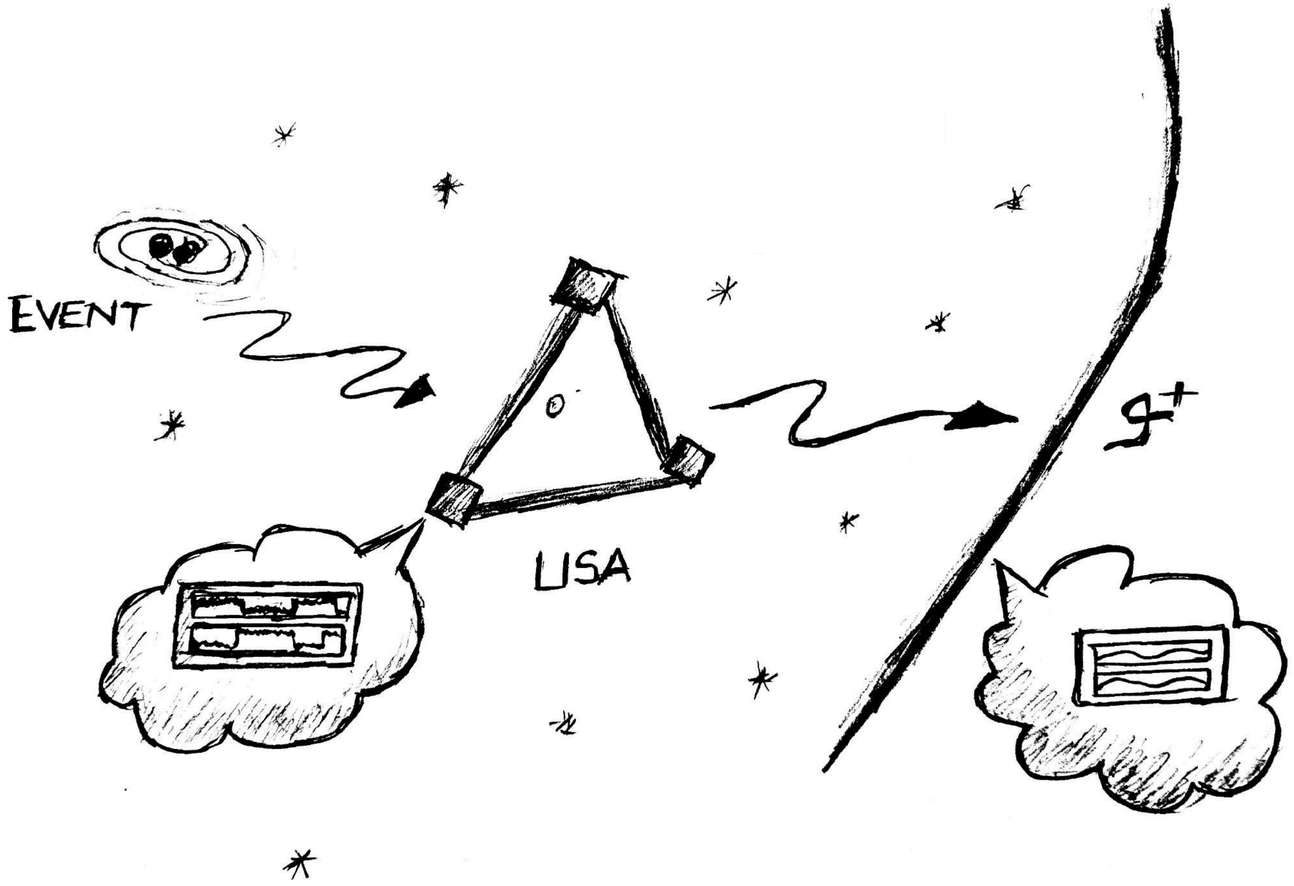
(i.e. an idealisation with epistemic control/de-idealisation procedure)

can be cast as

an inexact description of the system

(i.e. an approximation admitting an account of errors)

Is **Asymptotic Flatness** a Controlled Idealisation or Constitutive Element

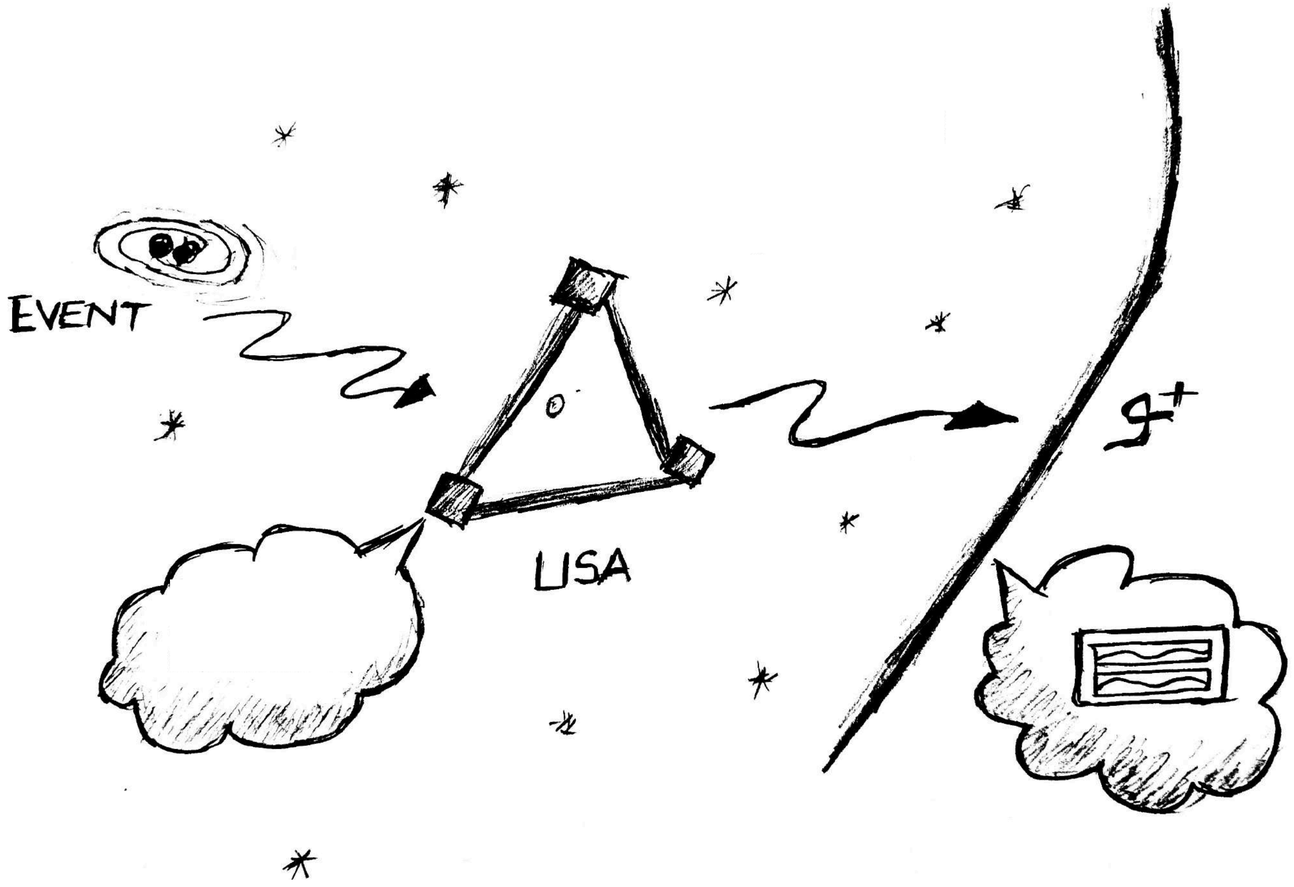


Constitutive Element

is a

precondition for the coherence and/or intelligibility (of a relevant concept)

Is **Asymptotic Flatness** a Controlled Idealisation or Constitutive Element



Constitutive Element

is a

precondition for the coherence and/or intelligibility (of a relevant concept)

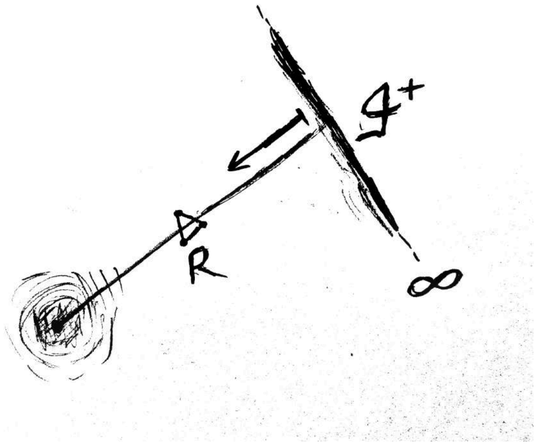
De-idealising

‘Null’ + ‘Infinity’

Not actually at infinity/boundary but…

De-idealising

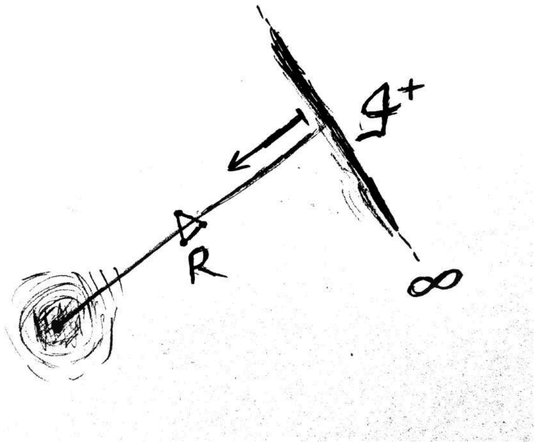
‘Null’ + ‘Infinity’



Not actually at infinity/boundary but **at some large distance on the bulk**

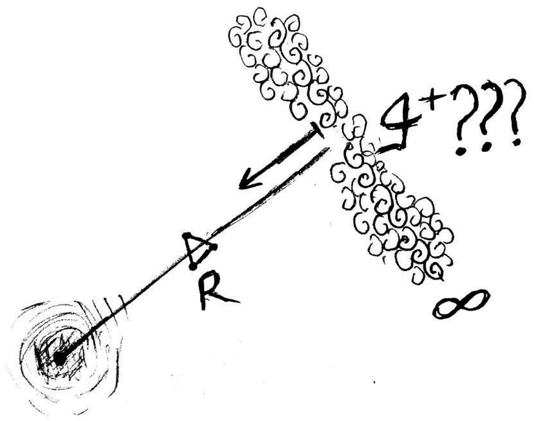
De-idealising

‘Null’ + ‘Infinity’



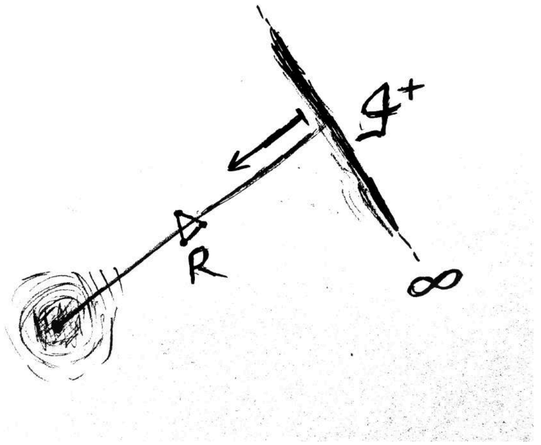
Not in asymptotically Minkowski (island universe) but…

Not actually at infinity/boundary but at some large distance on the bulk



De-idealising

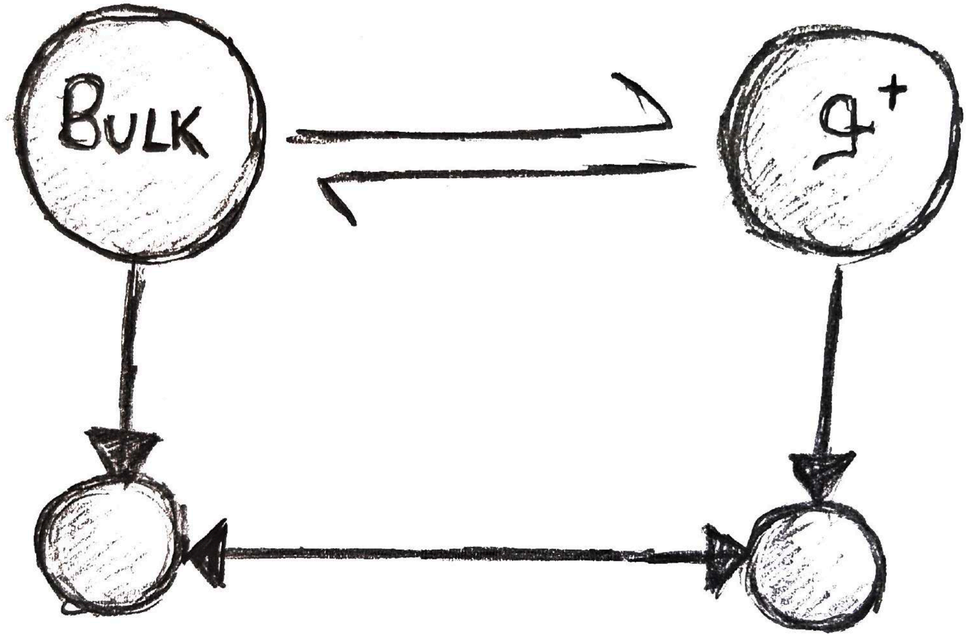
‘Null’ + ‘Infinity’



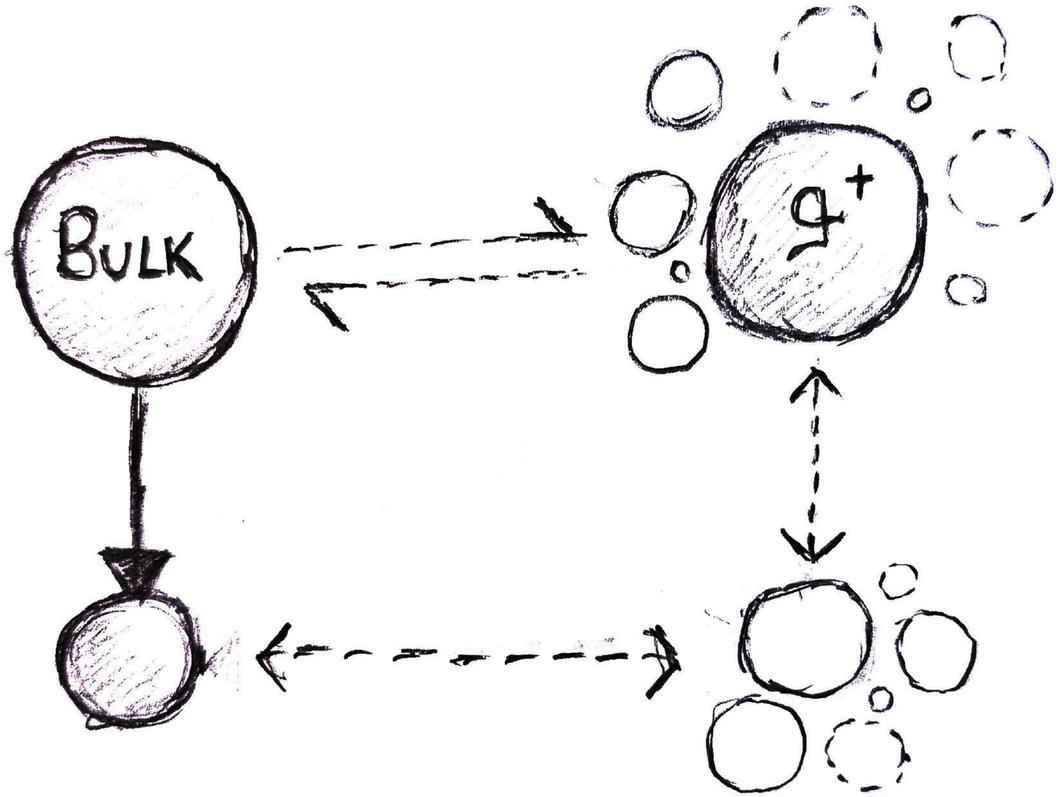
Not in asymptotically Minkowski (island universe) but asymptotically de Sitter (**cosmological principle + accelerated expansion**)

Not actually at infinity/boundary but at some large distance on the bulk

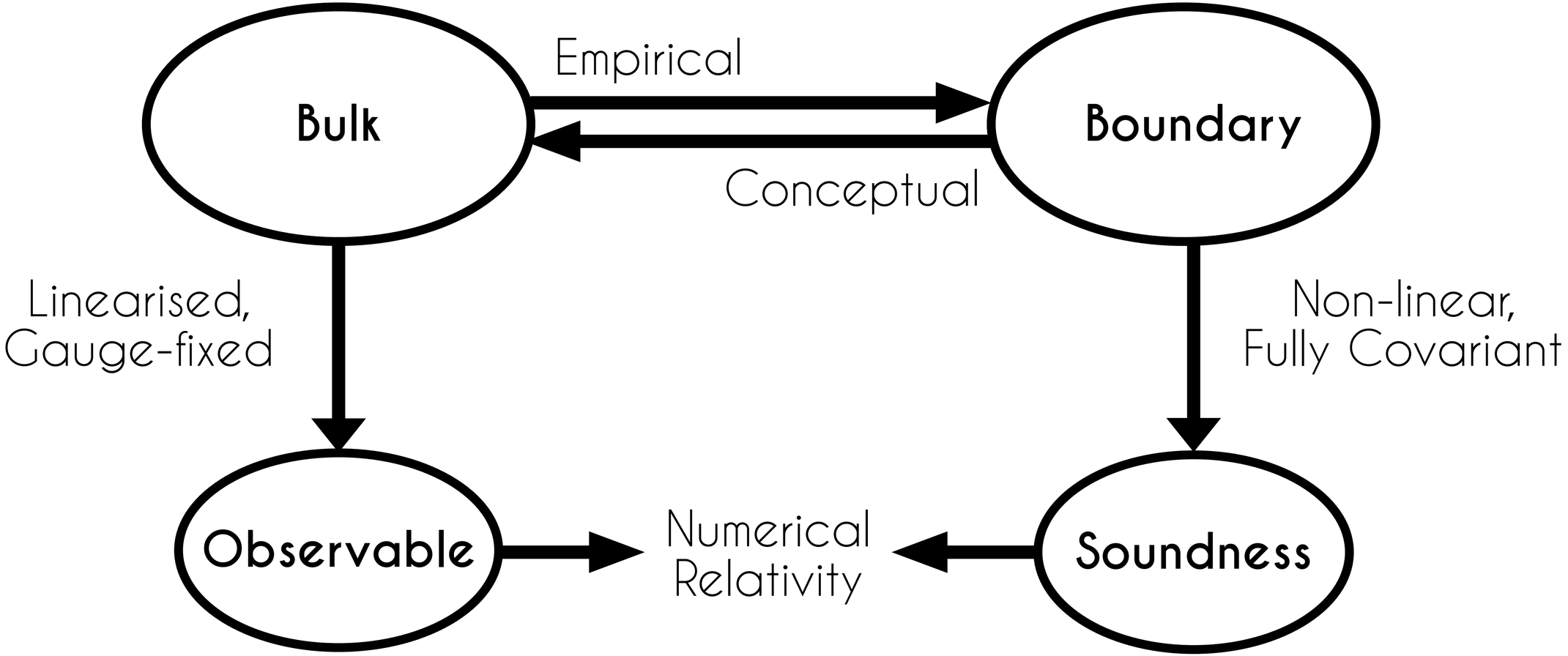
De-Idealising Infinity



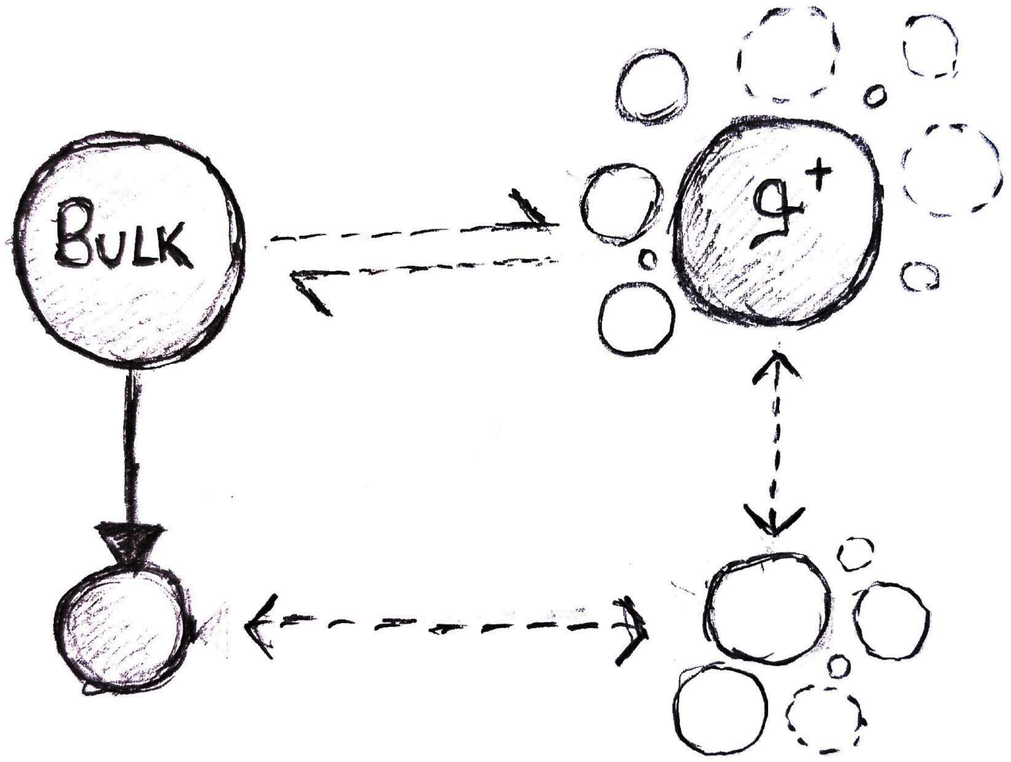
De-Idealising Null



Two Domains Interlocked

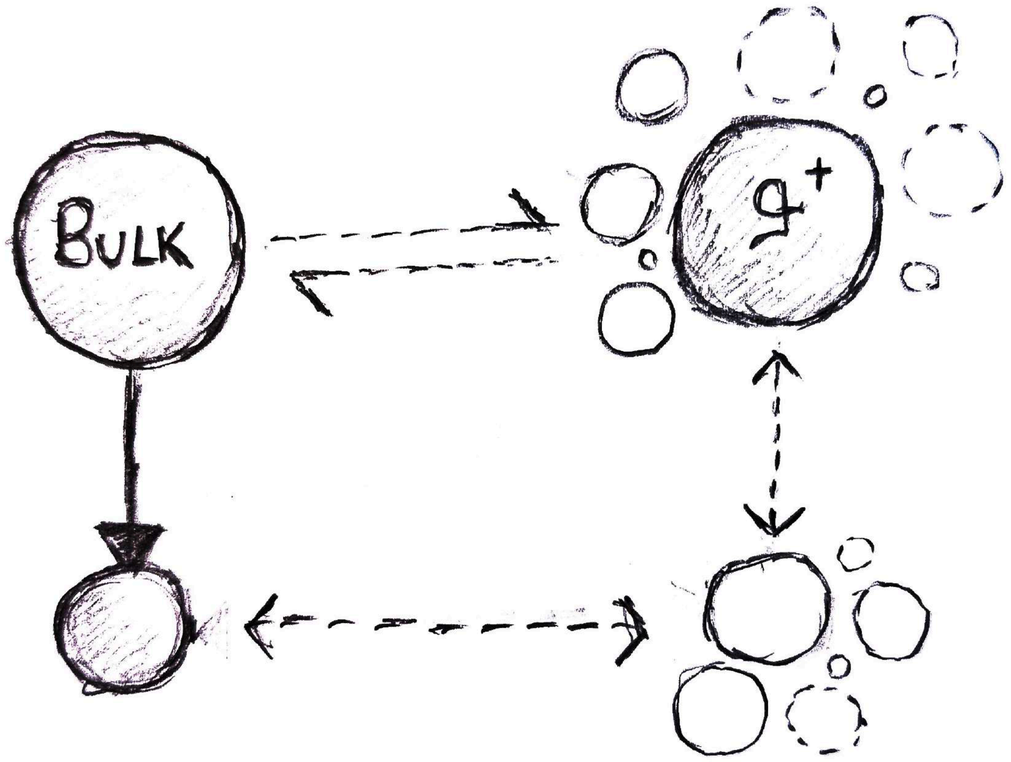


Two Domains in Cosmology



For a given observable, the general situation is -

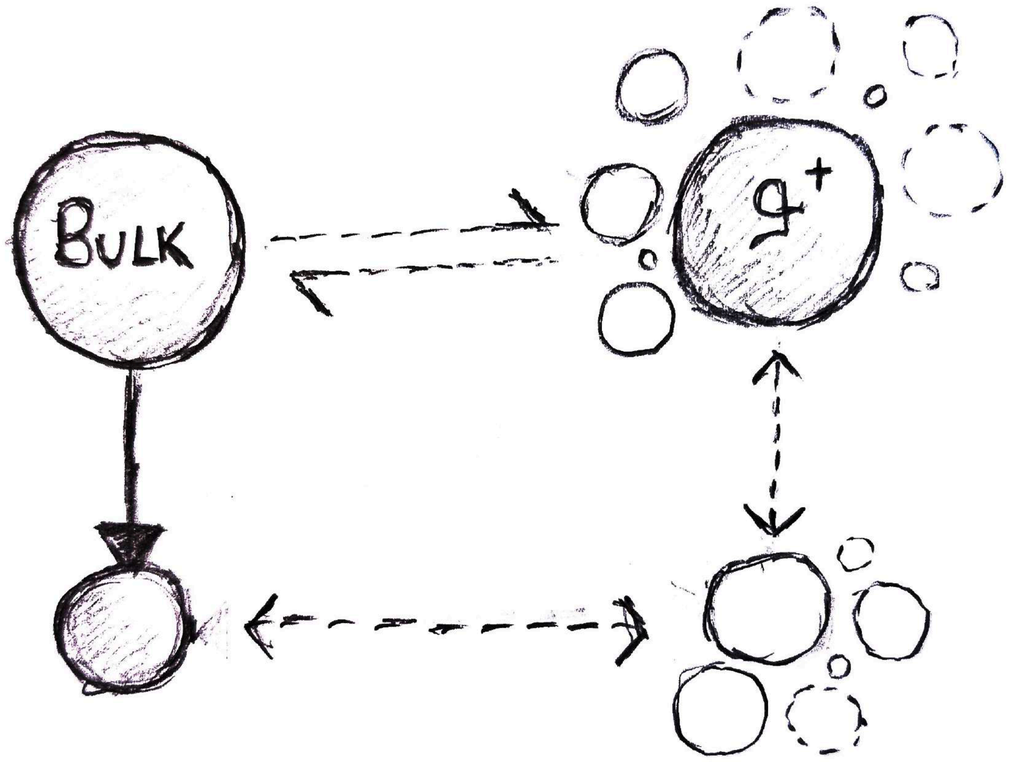
Two Domains in Cosmology



For a given observable, the general situation is -

Some results on the bulk,

Two Domains in Cosmology

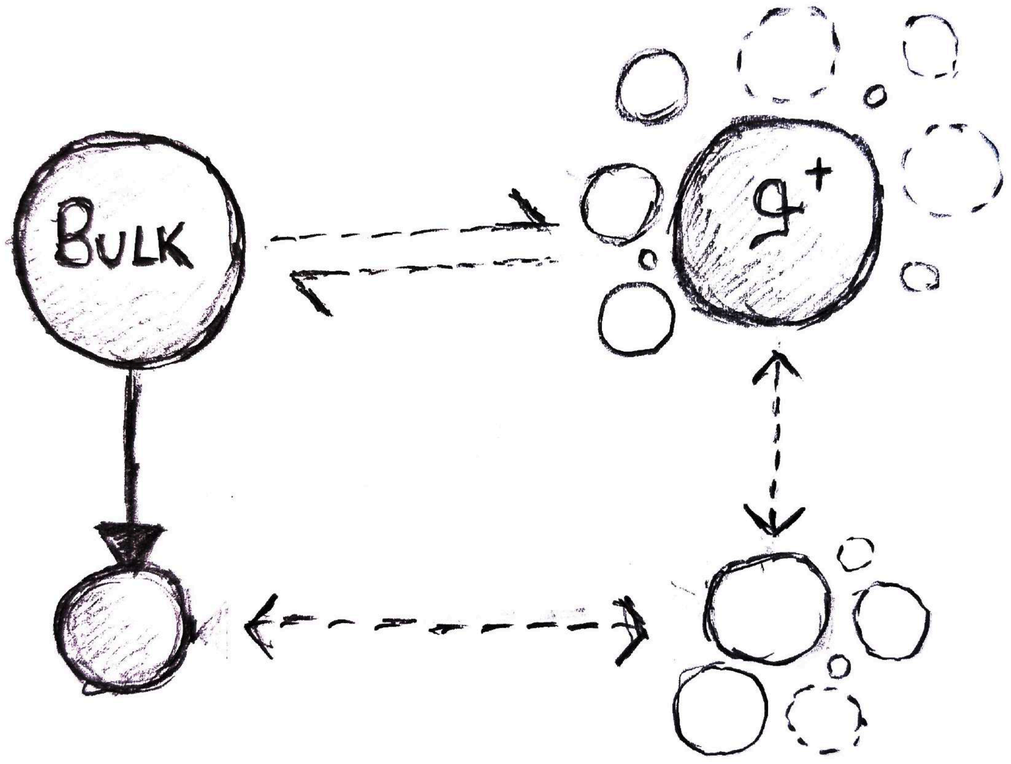


For a given observable, the general situation is -

Some results on the bulk,

many inequivalent attitudes towards the boundary

Two Domains in Cosmology



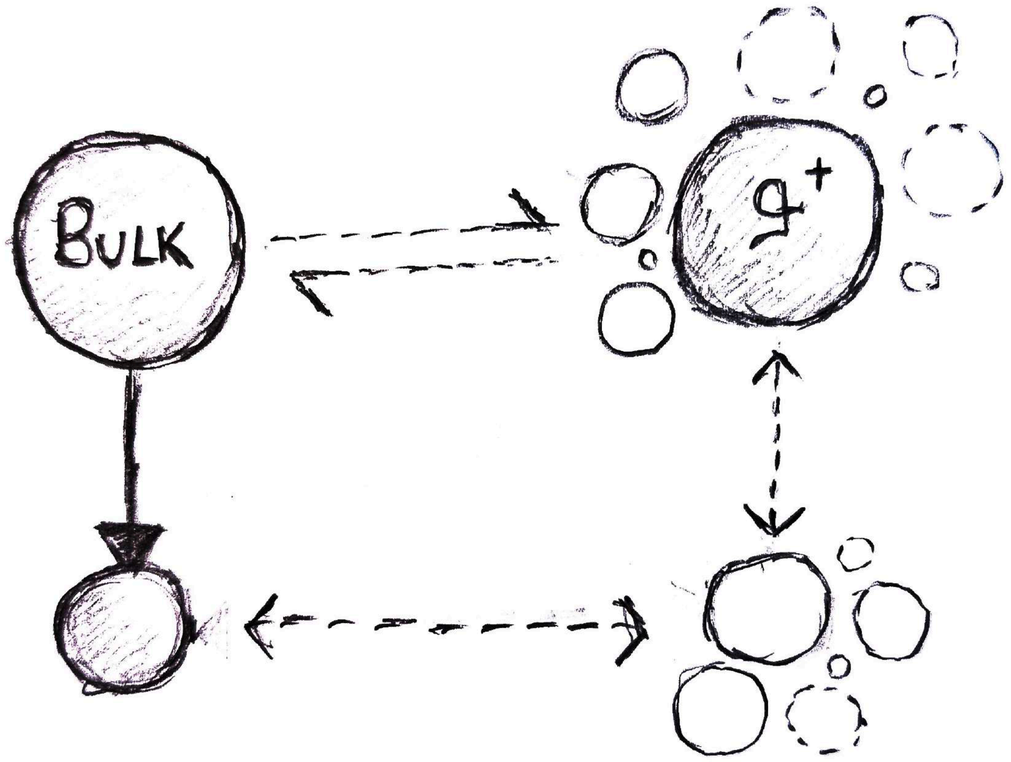
For a given observable, the general situation is -

Some results on the bulk,

many inequivalent attitudes towards the boundary

that may or may not admit target phenomena

Two Domains in Cosmology



For a given observable, the general situation is -

Some results on the bulk,

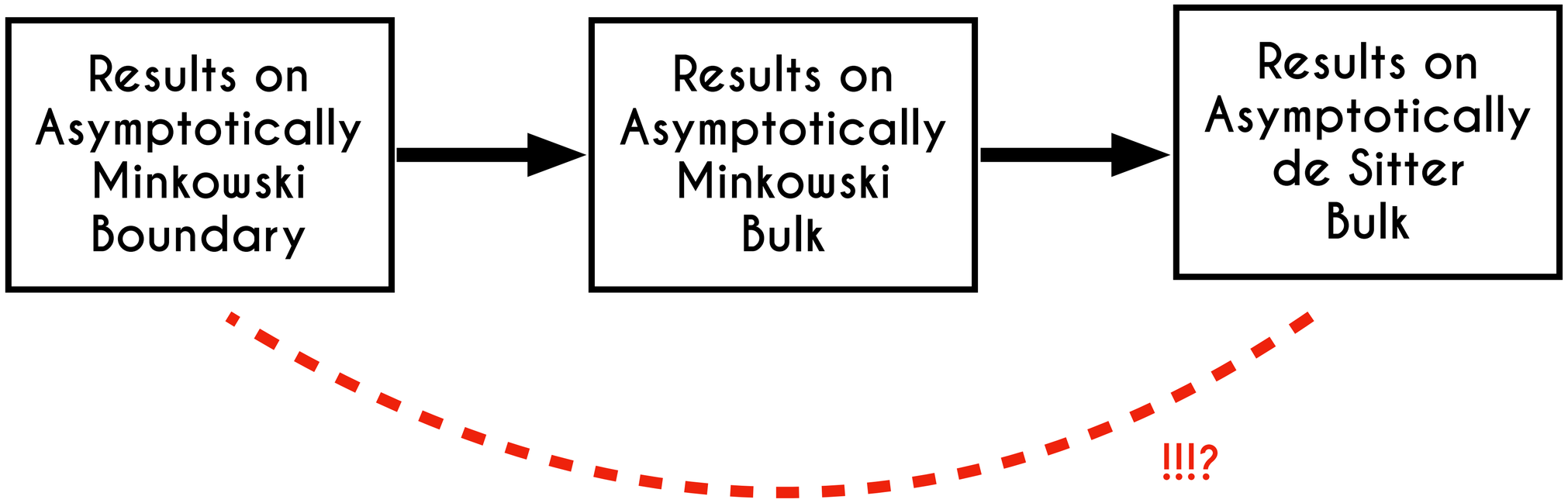
many inequivalent attitudes towards the boundary

that may or may not admit target phenomena

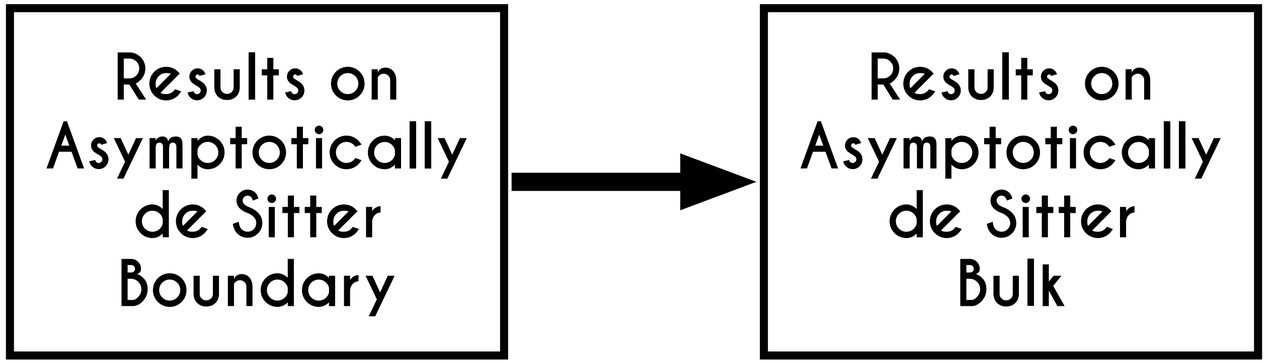
in a way that may or may not limit to the bulk results

Existing Support for Memory

Existing Support for Memory



Desired Support for Memory



Until then,

if one raises skepticism against the existence of gravitational wave memory in cosmological spacetimes

we do not have sufficiently a robust and satisfactory response

States of affairs infinitely far bear on understanding of local phenomena?!?!

Positive cosmological constant, no matter how small, revises our understanding of gravitational waves

Different aspects of null infinity suggest different approaches towards cosmological de-idealisation

Future observables provide impetus for this de-idealising pursuit

Some (modest) goals

Thank you and Happy Birthday Roger!

Acknowledgements

Abhay Ashtekar, Dennis Lehmkuhl, Henrique Gomes, Gal Ben-Porath, Jeremy Butterfield, Sam Fletcher, Bryan Roberts and Oxford Philosophy of Physics Graduate Community

www.kartiktiwari.com